
GDP AND TFP IN POWIATS OF THE ŁÓDZKIE VOIVODESHIP. ESTIMATION AND ANALYSIS OF DIFFERENTIATION

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Abstract: The main objective of the research was to estimate the level of GDP and total factor productivity (TFP) in the counties ('poviats') of the Łódzkie voivodeship in the period 2002-2019. The gross product in poviats was determined by disaggregating the GDP of the Łódzkie voivodeship in proportion to the revenues of poviat budgets from personal income tax PIT and to the shares of poviats in the voivodeship wage fund. TFP was determined on the basis of a labour productivity model derived from the Cobb-Douglas production function with the assumption of constant returns to scale. A spatial panel data model estimated by the maximum likelihood method was applied. The poviat of Łódź was identified as the upper outlier in terms of the level of gross product. The dynamics of poviat values of GDP was similar to the national one, but poviats with a much faster rate of growth were identified. The highest level of TFP was observed in the poviat of Łódź. Very high productivity was also characteristic for the two other cities with poviat status, especially Skierniewice. In the Łódzkie voivodeship there was a progressive polarisation in terms of TFP with two leading poviats. No spillover processes were found. The capital city of the voivodeship, being itself the upper outlier, therefore did not play the role of a growth centre. It was also found that a clearly defined profile of economic activity in the poviat is conducive to faster TFP growth.

Keywords: total factor productivity (TFP), GDP, poviat, spatial panel data model.

1. Introduction

Gross Domestic Product (GDP) is considered to be the key measure of economic growth. This is despite opinions being raised in recent years that it fails to capture many key aspects of the quality of life (e.g. healthcare and accessibility to social care

facilities for the disabled), as well as the distribution of income across society. In addition, the digital economy is not adequately reflected by GDP, yet it is an accurate and most comprehensive indicator of the total output in an economy (Samuelson and Nordhaus, 2010, p. 370).

The production factor that is gaining the most importance today is technical progress, as it is the factor that keeps the economy on the path to growth. The effect of technical progress is an increase in the stream of production, which does not result from an increase in factor resources but from the increase in efficiency of the production processes. The efficiency of improvements resulting from qualitative changes (more appropriate skills or embedded technologies) is reflected in an increase in total factor productivity (TFP). Numerous empirical studies (e.g. Easterly and Levine, 2000, pp. 177-219, Helpman, 2010, pp. 19-33, Hulten and Isaksson, 2007, pp. 1-46,) indicate that most of the observed variation in GDP per capita is due to variations in TFP. Total factor productivity is an unobservable variable, most commonly calculated based on production function as a residual value, or the so-called Solow residual, within a growth accounting procedure derived from neoclassical growth theories. GDP data are therefore needed to estimate TFP. The Central Statistical Office (GUS) provides data on the value of GDP for the whole economy and on the NUTS-2 and NUTS-3 levels. No information is available on GDP generated in NUTS-4 regions (poviats), although it is the poviats that can be considered as fundamental local economies identified with a given labour market (Zauchna et al., 2015, p. 248). The estimation of county (powiat) TFP values must therefore be preceded by an estimate of its GDP.

The aim of the study was to estimate GDP and TFP in the poviats of the Łódź voivodeship in 2002-2019, and to analyse the variation of these variables between the poviats in subsequent years.

Alternative ways of disaggregating voivodeship values of GDP to the powiat level were presented by Tokarski (2013), Ciołek and Brodzicki (2015), Dykas and Misiak (2018). These proposals are based on the determination of average shares of poviats in the voivodeship values of: gross fixed assets, sold production and investment outlays of enterprises or on the basis of revenue of powiat budgets from personal income tax (PIT) and from agricultural tax, or on the basis of the share of poviats in the voivodeship wage fund. In the presented study, a combination of the two latter methods was used. Estimates of GDP in the poviats of the Łódzkie voivodeship were based on the values of product generated there (NUTS-2 level), being primary data derived from source information. The GUS data for sub-regions (NUTS-3 level), which are the result of the division of voivodeship values, were not used in order not to accumulate subsequent disaggregation errors.

Empirical analyses aimed at estimating TFP have been conducted at different regional levels. Most studies concerned the national level, in total or by sections of the Polish Classification of Activities (PKD) (e.g. Górajski and Błażej, 2020; Świczewska, 2013) or international comparisons (e.g. Dańska-Borsiak, 2018;

Młynarzewska-Borowiec, 2018). In recent years, attempts to estimate and identify productivity determinants in voivodeships and poviats have become more frequent (e.g.; Bolińska, 2018; Ciołek and Brodzicki, 2015; Dańska-Borsiak, 2012; Dykas and Misiak, 2018; Tokarski 2010), reflecting a broader interest in analyses concerning smaller spatial units. In the presented study, the estimates of TFP were based on the two-factor neoclassical production function estimated from the spatial panel data.

The article is structured as follows. The second section contains a description of the data and methods used in the subsequent analysis. The empirical results are presented in the third section. First, estimates of gross product produced in poviats are presented and discussed. Then the results of the estimation of the productivity model, on the basis of which TFP was determined, are presented. An analysis of spatial and temporal diversification of total factor productivity was also carried out. The fourth section concludes.

2. Methods and data

According to the Methodological Description (the Central Statistical Office (GUS) website), GDP can be calculated in three ways:

(a) production approach: the sum of gross value added of institutional sectors or branches plus taxes on products less subsidies on products (which are not allocated to sectors or branches). It is also the balancing item of the production account of the country economy;

(b) expenditure approach: the sum of final use of goods and services (gross consumption and gross accumulation) by resident institutional units plus exports minus imports of goods and services;

(c) income approach: the sum of the outflows of the generation of income account of the total economy (remuneration of employees, taxes on production and imports less subsidies, gross operating surplus and mixed income of the total economy).

As poviats are not separate economies, there can be significant differences between GDP estimates based on each of these methods. The most important reason being that it is impossible to estimate the flows of goods, commodities and services between poviats.

The presented analysis was carried out on the basis of panel data for 24 poviats of the Łódzkie voivodeship for the period 2002-2019. The following variables (in constant prices) were used: *PIT* – tax revenues of poviat budgets from personal income tax, *AT* – tax revenues of poviat budgets from agricultural tax, *W* – average monthly gross wages and salaries, *LP* – number of employed persons according to LFS, *K* – gross value of fixed assets in enterprises. The variables *PIT*, *AT*, *W* and *LP* were used to estimate GDP, and the variables *LP* and *K* to estimate TFP in poviats.

Poviat GDP values were estimated by allocating the GDP of the Łódzkie voivodeship. In the first step, such an allocation was made based on two proposals contained in Ciołek and Brodzicki (2015): the shares of poviats in taxes constituting

income of the state budget from personal income tax (PIT) and agricultural tax, and the share of poviats in the voivodeship wage fund. Then, the final values of GDP in the poviats were calculated as a geometric mean of the values obtained in step one.

The estimation of GDP according to tax revenue is based on the assumption that taxes are related to the product generated in a region. PIT shares were used as the basis. The use of corporate income tax (CIT) would not be a good option because this tax is paid in the poviat (county) where the company's head office is located, but in practice activities may be carried out outside this poviat. Moreover, CIT payers benefit from tax exemptions or apply write-offs to cover losses from previous years. The calculation of GDP on the basis of PIT is also biased, but its magnitude is much smaller than if CIT were used. The bias is due to two reasons; firstly, income tax is not paid by persons engaged in agricultural activities, and secondly, some employees work (contribute to production) and earn income in other poviats than those they reside in, spend their earnings and pay income tax PIT. The problem of not paying PIT by those engaged in agricultural activity was solved using information about the revenue of poviats from agricultural tax¹. The percentage of voivodeship GDP equal to the percentage of GVA generated in the agricultural sector, was distributed among the poviats in proportion to their share in agricultural tax in subsequent years.

The estimation of GDP in the poviats on the basis of the wage fund was based on the assumption that the produced output is related to the remuneration of the factors that create it. The wage fund in poviats was calculated as a product of the average gross wage and the number of employees, and then the voivodeship GDP values were allocated according to the shares of poviat wage funds in the fund of the whole voivodeship. The disadvantage of this method is that the calculation of GDP is linked only to the remuneration of the labour factor, ignoring the remuneration of capital.

Finally, GDP values were determined as a geometric mean of the estimates obtained by the two methods described above. This approach is intended to reduce the bias of poviat GDP values arising when allocating the voivodeship GDP according to the share of poviats in revenues from PIT. The bias results from the fact that taxes are paid at the place of residence and not at the place of work where the product is actually produced.

The method of estimating TFP in poviats of the Łódzkie voivodeship was taken from (Tokarski , 2008). TFP was calculated from the productivity function resulting from the Cobb-Douglas production function with the assumption of constant scale effects. The function has the form:

$$y_{it} = \alpha_0 e^{gt} k_{it}^\alpha, \quad (1)$$

¹ The tax base for agricultural tax for land of agricultural farms depends on the area, types and classes of land and classification in the tax district. For other land it depends on the number of hectares. The tax is of little relevance to cities, but still some small part of their budget revenues comes from this tax. In Łódź, the percentage of GDP in section A to GDP outside this section is 0.08%, in two other cities with poviat (county) status it is around 0.3%.

where: $y_{it} = PKB_{it}/LP_{it}$ and $k_{it} = K_{it}/LP_{it}$ denote respectively: average labour productivity and capital per worker in poviats i and year t , g is the rate of technical progress in the Hicks sense, t is the time variable, and α is the elasticity of output with respect to changes in capital; value $\alpha_0 e^{gt}$ is interpreted as total factor productivity.

Poviats are relatively small spatial units whose administrative borders do not reflect the so-called functional areas, characterised by common socio-economic and spatial features. Therefore, values of a given variable in one poviat may influence its values in neighbouring poviats, which means that spatial relations in the model must be taken into account. For this reason, function (1) was estimated as a spatial panel data model. The classification and estimation methods for models of this class are discussed in detail in (Sucheck, 2012, Chapters 3 and 4). Of the alternative specifications, the best results were obtained for the Spatial Error Model with random effects (SE-FEM).

3. Empirical results

This section presents the estimates of GDP and, in the second part, the estimates of TFP for poviats of the Łódzkie voivodeship.

3.1. Estimates of GDP in poviats

The first stage of empirical research was to estimate the poviat values of GDP. The two alternative ways of allocating the voivodeship GDP values described in Section 2 were used. Then the final values of poviat GDP were calculated as their geometric mean.

The allocation of the GDP generated in the Łódzkie voivodeship in proportion to the revenue of the poviats from income and agricultural taxes was made according to the formula:

$$GDP_{p,t,tax} = \frac{PIT_{p,t}}{PIT_{v,t}} \cdot (1 - uGVA_{agr,t}) \cdot GDP_{v,t} + \frac{AT_{p,t}}{AT_{v,t}} \cdot uGVA_{agr,t} \cdot GDP_{v,t}, \quad (2)$$

where: $PIT_{p,t}$ and $PIT_{v,t}$ are the values of PIT tax, $AT_{p,t}$ and $AT_{v,t}$ are the values of agricultural tax paid to the poviat and voivodeship budgets respectively, $uGVA_{agr,t}$ is the share of voivodeship GVA produced in section A (agriculture, forestry, hunting and fishing); index t denotes the time period.

The first component of formula (2) describes the distribution of that part of voivodeship GDP which is equal to the share of GVA produced outside section A. This part was allocated proportionally to the shares of poviats in PIT. The part equal to the share of GVA (the second component of formula (2)) generated in section A was allocated proportionally to shares in the agricultural tax. The use of information about the poviats' income from agricultural tax results from the fact that it is the only tax paid

by persons engaged in agricultural activity (they do not pay PIT). From 2002 to 2019, on average, about 4% of the gross value added of the voivodeship was generated in the agricultural sector. This percentage was highest in 2007, when it amounted to 5.1%, but in the following years it decreased, and in 2019 was equal to 3.3%.

The correct determination of the level of personal income taxes for cities with poviat status requires a correction of the GUS data on the share of communes (gminas) shares in the state budget tax revenue. The data published by the office for land poviats concern only the revenue of gminas. For cities with poviat rights, on the other hand, they concern both revenues of gminas (communes) and poviats (counties); relying on these data would therefore result in an overestimation of the share of cities in generating GDP. The binding act on public finances (Ustawa z dnia 27 sierpnia 2009 r....) regulates the distribution of PIT revenues between poviats and gminas. The share of gminas is at 39.34% and the share of poviats 10.25% of the total PIT tax revenue in a given region. However, in reality the share of poviats is lower due to the statutory inclusion of additional indicators. The correction of the share of cities with poviat status (the city of Łódź, the city of Piotrków Trybunalski, the city of Skierniewice) was based on the procedure described by Wojnicka-Sycz (2013).

The estimation of GDP in the poviats on the basis of the wage fund was based on the assumption that the produced output is related to the remuneration of the factors that create it. Wage funds in the voivodeship and the poviats were calculated as a product of the average gross wages and the number of employed. Then, GDP in the voivodeship was distributed among the poviats in proportion to the shares of their wage funds in the voivodeship fund, using the following formula:

$$GDP_{p,t,wage} = \frac{W_{p,t} \cdot LP_{p,t}}{W_{v,t} \cdot LP_{v,t}} \cdot GDP_{v,t}, \quad (3)$$

where $W_{p,t}$ and $W_{v,t}$ are the average monthly gross wages and salaries, $LP_{p,t}$ and $LP_{v,t}$ are the number of persons employed in a given poviat and the voivodeship respectively; index t denotes the time period.

As expected, the GDP estimates obtained by the two methods differ significantly. The greatest differences were found in the Bełchatowski poviat, where the GDP values estimated on the basis of the wage fund were higher than those estimated on the basis of taxes over the whole period. This is the poviat in which a lignite mine is located, which offers relatively high wages that attract workers from neighbouring poviats. An interesting interdependence can be observed in Łódź and the neighbouring poviats (see Figure 1). In the latter, the GDP calculated on the basis of taxes was higher than that calculated on the basis of the wage fund, with the difference increasing systematically. The opposite trend was visible in the city poviat of Łódź, where in 2018 and 2019, GDP estimated on the basis of the wage fund even exceeded GDP on the basis of taxes. These trends can be justified by the process of outflow of inhabitants from Łódź to suburban areas who, however, commute to work in the city. Taxes are paid in the county of residence and not in the place of work where the product is actually produced. This thesis

is supported by data on the net migration rate published by the Central Statistical Office (GUS) since 2005. Net migration rate is a measure which shows what fraction of the population living in a given area are immigrants, and what fraction emigrated in a given period. For the poviats of Łódź the rate is negative in all the studied years (from -21.7 in 2007 to -5.7 in 2016), while for each of the neighbouring poviats it is positive (on average, the highest was in the Łódzki Wschodni powiat, approximately 80-90, and the lowest in the Pabianicki powiat, approximately 20-35).

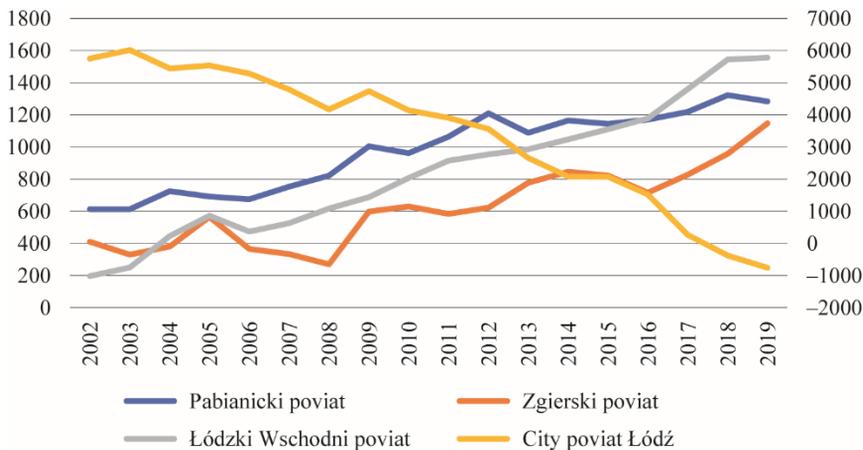


Fig. 1. The differences in $GDP_{p,t,tax} - GDP_{p,t,wage}$ in the city poviat of Łódź (right axis) and neighbouring poviats (left axis)

Source: own work.

None of the above ways of estimating GDP is immune to error. The main objections are indicated in section 2. Calculating GDP based on their result can, to some extent, reduce the bias of $GDP_{p,t,tax}$ resulting from working and paying taxes in different poviats. The Central Statistical Office publishes data on GDP in sub-regions. However, these are the result of a disaggregation of the original voivodeship data, so they were not used to allocate GDP to the poviats. Łódź is a separate sub-region, hence it was possible to compare the above estimates with the GUS data for the sub-region of Łódź. The GDP values determined on the basis of the wage fund proved to be underestimated and the values on the basis of taxes – overestimated in relation to the GUS information. In the search for the best way to calculate GDP by poviats as the resultant of tax and wage GDP, alternative averages of these values were calculated for the poviat of Łódź. The results closest to the GUS information were obtained on the basis of the geometric mean:

$$GDP_{p,t} = \sqrt{GDP_{p,t,tax} \cdot GDP_{p,t,wage}} \quad (4)$$

The GDP values calculated on the basis of formula (4) for all poviats over the period 2002-2019 became the basis for subsequent analyses.

Quartile groups for the first and last year of analysis are shown in Figure 2. The upper outliers are poviats in the fourth quartile group, for which GDP is more than 1.5 times the interquartile range above the value of the third quartile.

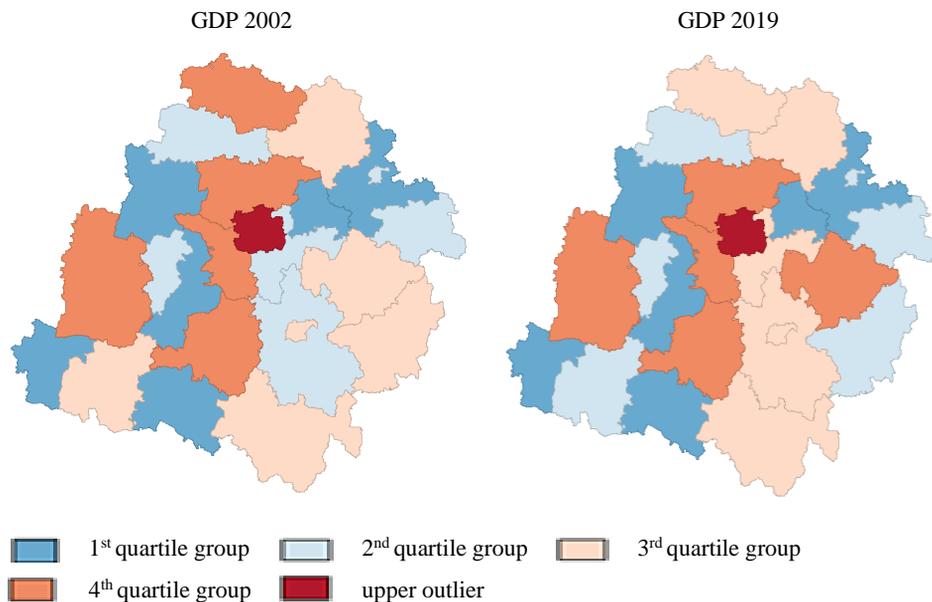


Fig. 2. Spatial differentiation of GDP in poviats in 2002 and 2019

Source: own work.

Table 1 presents the poviat GDP values in the first and the last year of the analysis, membership in the quartile groups indicated in Figure 2, together with an indication of the change in the ranking position and the mid-period pace of change.

The list of poviats belonging to the lowest, fourth group did not change, and among the poviats of the highest group, only Kutnowski dropped by one position in 2019. The biggest changes occurred in the two middle groups. When analysing the results in Table 1, it must be remembered that the GDP are not per capita values. Their variation is very high, with the coefficient of variation of about 172%, and in the poviat of Łódź, indicated in Figure 1 as the upper outlier, about 38% of the voivodeship GDP was produced. However, these values cannot be directly interpreted as reflecting the level of wealth, as for this purpose one would have to use per capita values.

Table 1. GDP in poviats in 2002 and 2019; in mln PLN, at constant prices, 2002 = 1

Name of poviat	Quartile group		GDP (in mln. PLN)		rank change*	T_n (w %)*
	2002	2019	2002	2019		
Łódź	4	4	19170.51	36553..98	0	3.87
Bełchatowski	4	4	3284.68	5819.23	0	3.42
Zgierski	4	4	2398.07	5582.56	0	5.10
Pabianicki	4	4	2072.47	3953.39	1	3.87
Sieradzki	4	4	2147.93	3876.69	-1	3.53
Tomaszowski	3	4	1834.48	3675.94	1	4.17
Kutnowski	4	3	1927.29	3359.40	-1	3.32
Radomszczański	3	3	1763.26	3352.23	0	3.85
Piotrków Trybunalski	3	3	1732.79	3037.74	0	3.36
Piotrkowski	2	3	1152.32	2637.13	5	4.99
Łowicki	3	3	1529.45	2576.44	-1	3.12
Łódzki Wschodni	2	3	981.69	2496.73	4	5.64
Wieluński	3	2	1249.81	2360.24	-1	3.81
Opoczyński	3	2	1518.72	2104.89	-3	1.94
Zduńskowolski	2	2	1221.54	1981.24	-2	2.89
Skierniewice	2	2	1154.39	1905.72	-2	2.99
Rawski	2	2	857.41	1761.22	1	4.33
Łęczycki	2	2	896.96	1631.83	-1	3.58
Pajęczański	1	1	739.84	1542.32	1	4.42
Łaski	1	1	777.82	1415.58	-1	3.59
Poddębicki	1	1	588.62	1401.33	1	5.23
Wieruszowski	1	1	660.46	1303.54	-1	4.08
Skierniewicki	1	1	543.26	1244.67	0	5.00
Brzeziński	1	1	411.22	862.83	0	4.46

* Rank change is the difference between the ranking position in 2019 and in 2002; T_n denotes average growth rate calculated as $\bar{T}_n = (\bar{t}_G - 1) \cdot 100\%$, where $\bar{t}_G = \sqrt[n-1]{x_T/x_1}$.

Source: own work.

Gross regional product showed a growing tendency in all poviats. The dynamics of GDP changes in the counties (poviats) between 2002 and 2019 was varied and strongly linked to the national dynamics. Output growth was highest in 2006 and 2007 and lowest in 2012-2013. The mid-period pace of change T_n for the whole voivodeship amounted to 3.86%. The fastest pace of change T_n exceeding 5% was observed in the following poviats: Łódzki Wschodni, Poddębicki, Zgierski and Skierniewicki. In the

Poddębicki powiat, in the period under analysis, a thermal, pool and sports complex called Termy Uniejów was built, managed by a municipal company. Its creation and ongoing expansion attracts numerous investors and tourists. In Stryków, located in the Zgierski powiat, there is a junction of the A1 and A2 motorways. Due to this location, Stryków attracts numerous investors, especially from the logistics sector. Although these investments are not directly connected to technology, the TFP growth may come from a public investment to establish a cluster, which appears to be successful. A logistics sector is also developing in the neighbouring powiat Łódzki Wschodni, while the Skierniewicki powiat is an important and developing centre of the food processing industry.

3.2. Estimates of TFP in poviats

TFP values determined on the basis of productivity function of formula (1) are affected by the way capital and labour inputs are measured, and by whether additional production factors, primarily human capital, are taken into account (Florczak, 2007; Świczewska, 2007, pp. 74-77; Welfe, 2001, pp. 112-188, 2003). Measurement of human capital at powiat level is impossible due to the unavailability of statistical data, so a two-factor model was estimated. Due to the possibility of spatial relations between variables, a spatial panel data model based on function (1) was applied. Three types of models were used: Spatial Error Model (SEM), Spatial Lag Model (SLM) and Spatial Durbin Model (SDM), all of them with fixed effects (FE) and with random effects (RE). The best results (in the sense of statistical and substantive properties of the model) were obtained for the panel data random effects SEM model, which takes the estimation form

$$\ln(y_{it}) = \alpha_0 + \mathbf{g}^T \mathbf{t} + \boldsymbol{\alpha}^T \ln(\mathbf{k}_{it}) + (\mu_i + u_{it}), \quad u_{it} = \lambda(\mathbf{W}\mathbf{u})_{it} + \varepsilon_{it}, \quad (5)$$

where parameter λ captures a spatial autocorrelation effect of the error term and $(\mathbf{W}\mathbf{u})_{it}$ reflects the spatial autoregressive process in errors according to which unobservable shocks affecting individual i interact with shocks affecting the said individual's neighbours. \mathbf{W} is the first order contiguity-based spatial weight matrix. The elements of the vectors $\ln(\mathbf{k}_{it})$ and \mathbf{t} are products of the dummy variables identifying the powiat type with, respectively: the log of capital per worker and the time variable t . The distinction was made because the assumptions about the output elasticity of capital and the rate of technical progress being equal for all poviats (counties) seem disputable. Three types of poviats have been distinguished: (1) land poviats, (2) cities with powiat status: Skierniewice and Piotrków Trybunalski and (3) city with powiat status Łódź. The latter was indicated as a separate type because it is the only metropolitan area in the voivodeship (ESPON, 2004, p. 90; *Koncepcja...*, 2012, p. 192). An attempt was made to distinguish the poviats surrounding the metropolis as a separate group, but it was later abandoned as it led to the highest TFP values in the land poviats, which seems incorrect.

Table 2. Estimation results of the labour productivity model

Regressor	city poviat Łódź*	other city poviats*	land poviats*
$\ln k$	0.3496	0.3846	0.4216
t	0.0222	0.0217	0.0263
α_0	3.2013	3.2013	3.2013
$\hat{\lambda}$	0.7278		
Wald test of spatial term λ	388.82	p -value	0.000
Pseudo R ²	0.834		

* p -value for all the parameter estimates is 0.000.

Source: own work.

According to the results in Table 2, the estimated rate of technical progress in the Hicks sense ranges from about 2.2% to 2.6%, and the elasticity of production with respect to capital from 0.35 to 0.42, depending on the type of poviat. Such values seem acceptable. The literature usually assumes a rate of technological progress of 2% (e.g. Świeczewska, 2013) and an elasticity of output with respect to capital of 0.3 to 0.6 (e.g. Ciołek and Brodzicki, 2015; Gosińska and Ulrichs, 2020; Mankiw, Romer, and Weil, 1992), depending on how human capital is treated. The smallest impact of capital on the value of GDP was found in the city of Łódź, and the largest in the land poviats. Assuming constant returns to scale, this means that in city poviats, especially in Łódź, labour input has a greater impact on output than in land poviats. This seems justified by the larger share of the service sector, and probably also a higher level of human capital in the city poviats. The rate of technological progress is slightly higher in land poviats, which may result from its lower initial level and be a result of the social policy of the central government.

The estimated value of the spatial autocorrelation parameter λ is large and significant, which means that some determinants of production omitted from the model are spatially autocorrelated, and unobserved shocks follow a spatial pattern. Moreover, the significance of all the variables and the high value of the pseudo R² coefficient allowed to consider that the TFP values calculated from the estimates of the parameters of model (5) based on the following formula (6) are reliable:

$$\ln(TFP_{it}) = \ln(\hat{y}_{it}) - \hat{\alpha}_p \ln(k_{it})_p \quad (6)$$

$(\hat{\alpha})_p$ and $\ln(k_{it})_p$ are the elements of vectors $\hat{\alpha}^T$ and $\ln(\mathbf{k}_{it})$ corresponding to poviat type p ($p = 1, 2, 3$).

The map in Figure 3 indicates poviats where TFP values belong to consecutive quartile groups and upper outliers (poviats where TFP exceeds the third quartile by more than 1.5 times the interquartile range). A ranking of poviats according to the level of TFP in the first and last year of analysis and the change in position is given in Table 3.

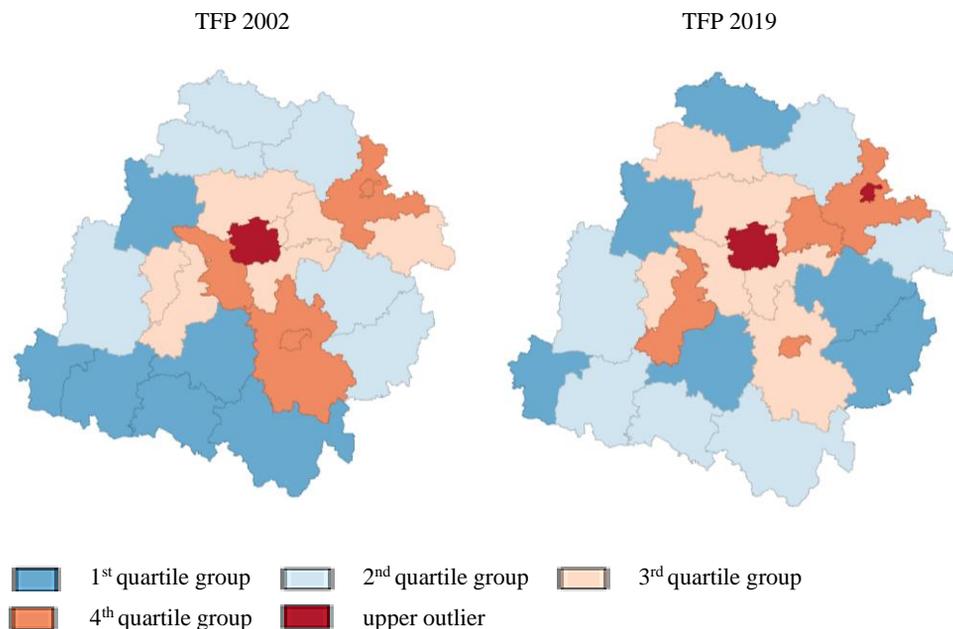


Fig. 3. Spatial differentiation of TFP in poviats in 2002 and 2019

Source: own work.

Table 3. Ranking of poviats by TFP: 2002 vs. 2019

Name*	Quartile group		Rank			Type of poviat**
	2002	2019	2002	2019	change	
1	2	3	4	5	6	7
Łódź	4	4	1	1	0	agglom
Skierniewice	4	4	2	2	0	service
Piotrków Trybunalski	4	4	4	3	1	service
<u>Brzeziński</u>	3	4	11	4	<u>7</u>	agr
Skierniewicki	4	4	3	5	-2	agr
<u>Łaski</u>	3	4	12	6	<u>6</u>	mixed
Łódzki Wschodni	3	3	7	7	0	service
Piotrkowski	4	3	6	8	-2	mixed
Zgierski	3	3	8	9	-1	ind
<i>Pabianicki</i>	4	3	5	10	-5	ind
<u>Łęczycki</u>	2	3	16	11	<u>5</u>	agr
Zduńskowolski	3	3	10	12	-2	ind
<i>Rawski</i>	3	2	9	13	-4	agr

Table 3, cont.

1	2	3	4	5	6	7
<u>Wieluński</u>	1	2	22	14	<u>8</u>	agr
Łowicki	2	2	13	15	-2	agr
Sieradzki	2	2	14	16	-2	mixed
<u>Pajęczański</u>	1	2	21	17	<u>4</u>	agr
Radomszczański	1	2	20	18	2	mixed
<i>Tomaszowski</i>	2	1	15	19	-4	service
<u>Wieruszowski</u>	1	1	24	20	<u>4</u>	agr
<i>Opoczyński</i>	2	1	18	21	-3	mixed
<i>Kutnowski</i>	2	1	17	22	-5	mixed
Bełchatowski	1	1	23	23	0	ind
<i>Poddębicki</i>	1	1	19	24	-5	mixed

* Underlining indicates names of poviats with the biggest increase, while italics a decrease of their ranking position.

** The poviat type was distinguished by the dominant type of economic activity: “service”, “ind”, “agr” denote respectively the dominant role of services, industry and agriculture; “mixed” denotes poviats with no specific profile; “agglom” is the centre of agglomeration. The method of determining a poviat type is described in detail in Laskowska and Żółtaszek (2021).

Source: own work.

Figure 2 shows that there were significant changes in the spatial distribution of TFP over the analysed period. The leading position was maintained by the poviat city of Łódź, with TFP exceeding the third quartile not only by more than 1.5, whilst even by more than three times the interquartile range both in 2002 and 2019. Another upper outlier, the poviat city of Skierniewice, emerged in 2019.

The dominant position of the city poviat of Łódź in terms of TFP, apart from the effect of agglomeration and academic potential, was also due to the presence of numerous enterprises from the household appliances and IT industries, characterised by high innovation potential. In Skierniewice and the surrounding poviat there are modern food processing plants, whose potential meant that the level of TFP in the poviat exceeded the third quartile by more than 1.5 times the interquartile range.

According to Figure 2 and the results in Table 3, in 2019 the first quartile group included all city poviats and two poviats in which agricultural activity is predominant: Skierniewicki and Brzeziński. Despite the fact that in their area agriculture remained a dominant activity, a decrease in the number of employees in this sector was noticeable, mainly in favour of the food processing industry (Skierniewicki) or services (Piotrkowski). This may indicate the growing efficiency and modernisation of agriculture, which requires less labour input. The Łęczycki, Wieluński, Wieruszowski and Pajęczański poviats, which recorded the largest increases in the

TFP ranking (Table 3), while remaining poviats with a dominant agricultural activity, were also characterised by a decrease in the number of people working in this sector. In Łęczycki and Pajęczński have a developing food industry, while in Wieluński – the production of technological equipment and steel structures. In the Wieruszowski powiat, situated by the border of the Łódzkie and Wielkopolskie voivodeships, there are numerous modern furniture industrial plants, which sell their products on the domestic market and abroad. In the group with lower values of total factor productivity, included in the third or fourth quartile group, poviats with mixed economic profile prevail (Table 3). As many as seven of them recorded drops in the ranking position, by two to five places. It seems, therefore, that the lack of a clear specialisation may be a factor unfavourable to technical progress., whereas new economic clusters boost the growth of some poviats.

Table 4 contains descriptive statistics for the extreme years of the analysed period. An increase in the average value of TFP with a simultaneous increase in variance and asymmetry of the distribution was apparent. This means that the number of poviats with low productivity has grown, and the increase in the average value results from an improvement in productivity in the strongest poviats. Thus, in the Łódź voivodeship there is a progressive polarisation with a distinct group of leaders (the poviats of Łódź and Skierniewice).

Table 4. Descriptive statistics: TFP in 2002 and 2019

	mean	Me*	s*	V*	skewness*	min	Max	range
2002	20.74	19.98	4.68	22.57	0.49	14.30	38.47	24.17
2019	34.82	32.88	8.50	24.40	0.69	24.97	62.54	37.57

* Me denotes the median, s – standard deviation, V – coefficient of variation. Skewness is calculated as the Pearson's median coefficient of skewness: $A_M = 3(\bar{x} - Me)/s$.

Source: own work.

However, as confirmed by the results of Moran's spatial autocorrelation test and local LISA statistics, there were no particularly strong spatial relations in the voivodeship. While in 2002 there was a cluster of high TFP values with the centre in Łódź, and a cluster with low values around the Sieradzki, Wieluński and Pabianicki poviats, in 2019 the LISA values were statistically insignificant. Moran's I statistics for the test of global spatial autocorrelation was equal to 0.245 in 2002, and 0.153 in 2019. The test was based on a null hypothesis of spatial randomness and pseudo *p*-values were respectively: 0.008 and 0.066. The results mean in particular that in Łódź voivodeship the processes of the spilling-over of the high values of total productivity to neighbouring poviats became weaker over time. While in 2002, Łódź was the centre of a high-high cluster, by 2019 Łódź and the two other cities with powiat status, although themselves characterised by high total productivity, were not playing the role of growth centres.

4. Conclusion

The conducted research aimed to estimate GDP and TFP in the districts of the Łódzkie voivodship and to analyse the variation of these variables in space and time. The gross product in poviats was determined by disaggregating the GDP of the voivodeship in proportion to the revenues of poviat budgets from personal income tax PIT, and to the shares of poviats in the voivodeship wage fund. It appears that the combination of these two approaches gave the possibility of a more accurate estimation of real GDP values by poviats. Estimates of GDP based solely on the wage fund did not take into account the contribution of capital to output creation. Another issue not addressed in both considered approaches was that personal income taxes are paid at the place of residence, which may be different from the place of work, and hence the production's creation. The intention of combining the two approaches was therefore to correct these differences. The resulting estimates of GDP in poviats were then used to estimate total factor productivity.

Throughout the whole period under examination, TFP values in poviats were increasing, with the dominant position of the poviat of Łódź being maintained. A high level of TFP was also found in other poviats with city status, namely Skierniewice and Piotrków Trybunalski. Generally, however, there were significant changes in the spatial distribution of TFP over the analysed period. Statistical analysis of the results indicated increasing differentiation, with a growing number of poviats with TFP values below the average and the fastest productivity growth in leading poviats, primarily in poviats with city status, i.e. Łódź and Skierniewice, in which TFP was much higher than in other poviats throughout the whole period. Thus in the Łódzkie voivodship there was a progressive polarisation with two leading poviats. No spillover processes were found. While in 2002 Łódź was the centre of a high-high cluster, by 2019 Łódź and the two other cities with poviat status, although themselves characterised by high total productivity, were not playing the role of growth centres. Total factor productivity increased primarily in poviats with a more clearly defined economic profile. Such regions are able to attract investments requiring the development of modern technologies and knowledge potential. These branches in the Łódzkie voivodeship are: household goods production, IT, the precision industry, biotechnology, modern food processing industry, and shared services centres operating as part of business process outsourcing. Their development is related to the importance of human capital.

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PKB I TFP W POWIATACH WOJEWÓDZTWA ŁÓDZKIEGO. OSZACOWANIE I OCENA ZRÓŻNICOWANIA

Streszczenie: Zasadniczym celem badań było oszacowanie poziomu PKB i łącznej produktywności czynników produkcji (TFP) w powiatach województwa łódzkiego w latach 2002-2019. PKB oszacowano, dezagregując PKB województwa proporcjonalnie do dochodów podatkowych powiatów oraz do ich udziałów w wojewódzkim funduszu płac. TFP wyznaczono na podstawie modelu produktywności pracy wynikającego z funkcji produkcji Cobba-Douglasa z założeniem stałych efektów skali. Zastosowano panelowy model przestrzenny estymowany Metodą Największej Wiarygodności. Powiat miasto Łódź zidentyfikowano jako region o zdecydowanie najwyższym poziomie wytworzonej produkcji. Dynamika powiatowych wartości PKB była zbliżona do ogólnokrajowej, ale wskazano powiaty o znacznie szybszym tempie wzrostu. Najwyższym poziomem TFP charakteryzował się powiat m. Łódź. Bardzo wysoka produktywność charakteryzowała też dwa inne powiaty grodzkie, zwłaszcza Skierniewice. W badanym okresie rosła polaryzacja powiatów ze względu na poziom TFP oraz nie występowały zależności przestrzenne. Stolica województwa, będąca *upper outlier*, nie pełniła więc funkcji ośrodka wzrostu. Stwierdzono również szybszy wzrost TFP w powiatach o wyraźniej określonym profilu działalności gospodarczej.

Słowa kluczowe: łączna produktywność czynników produkcji (TFP), PKB, powiat, przestrzenny model danych panelowych.