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THE BASE IN THE COMPUTATION OF *DFL*

Abstract: The degree of financial leverage *DFL* is one of the most popular leverage ratios used in literature. As an elasticity ratio, *DFL* informs about the scale of net profit relative (percentage) reaction to a 1% change in operating income. This paper focuses on the base value of profit necessary for the computation of *DFL*. The issue is important as the choice of the base numbers determines the value of *DFL*. It is argued in the paper that *DFL* should be regarded merely as a language convention, which communicates the changes in profitability. As such *DFL* can be sometimes useful as a tool in financial analysis. It is the discretionary nature of the base selection which subsequently determines the value of *DFL*, which, among other arguments, suggests that *DFL* should not be regarded as a risk measure in the sense used by the modern finance theory.

Keywords: financial leverage, degree of financial leverage, financial risk.

1. Introduction

The degree of financial leverage (*DFL*) calculates the relative change in earnings after taxes (*EAT*) caused by a 1% change in operating profit (earnings before interest and taxes, *EBIT*).

$$DFL = \frac{\%\Delta EAT}{\%\Delta EBIT} = \frac{\dfrac{EAT_1 - EAT_0}{EAT_0}}{\dfrac{EBIT_1 - EBIT_0}{EBIT_0}} = \frac{EBIT_0}{EBIT_0 - Int} = \frac{EBIT_0}{EBT_0}\ , \qquad (1)$$

where *Int* denotes fixed financial costs, i.e. the total amount of interest charged, while EBT_0 is earnings before taxes.[1] Subscript 0 denotes what from now on is referred to as a base or a benchmark – a profit number against which percentage changes in earnings are calculated. Subscript 1 is used to point to a profit number, which describes the level of profit the base changes to.

[1] For (1) to be true, the effective tax rate is assumed to be equal to the marginal tax rate (see [Dilbeck 1962]).

If $EBIT_0 > Int > 0$, the degree of financial leverage is greater than 1, i.e. the relative change in *EAT* is greater that the relative change in *EBIT*, hence the use of "leverage" concept seems fully justified. Financial activity of a company, i.e. taking debt and paying interest against it, makes the firm to pay fixed amount of interest, which in turn magnifies ("levers" or "gears up") *EAT* reaction to a relative change in *EBIT*.

The interpretation of *DFL* as an elasticity measure is straightforward, however, the meaning of the base, i.e. $EBIT_0$ and EAT_0, is less obvious. Are they actual historic numbers reported in the profit and loss account or the future forecasted profits, or maybe required/expected by management and/or investors levels of profit? How much discretion do analysts have in selecting the base for *DFL* calculation? Are all levels of *EBIT* and *EAT* legitimate? This paper attempts to provide answers to these questions by studying the nature of both the base numbers in *DFL* computation as well as the nature of *DFL* itself. The problem is important as *DFL* seems very popular, among academic scholars in particular, yet the definition of the base numbers differs from one author to another or is simply not provided.

2. *DFL* calculation for different bases

Let us start with a simple numerical example. Let operating income $EBIT_0$ amount to 100, interest paid $Int = 20$ and tax rate $T_0 = 19\%$. Hence it follows that $EAT_0 = (100–20) \times (1–19\%)=64.8$ and the degree of financial leverage is $DFL = 100/80 = 1.25$. The firm that contemplates, the impact of, say, a 20% rise in *EBIT*, i.e. from $EBIT_0 = 100$ to $EBIT_1 = 120$, on the size of *EAT*, must conclude that the net profit increases by 25% from 64.8 to $EAT_1 = (120–20) \times (1–19\%) = 81$. This is precisely the growth predicted by the level of *DFL*, i.e. 20% × 1.25 = 25%.

Little would change, if the scenario is different than 20% increase in *EBIT*. For 50% growth in *EBIT*, the increase in *EAT* is 62.5% (from 64.8 to 105.3), again in line with *DFL* = 1.25, i.e. 50% × 1.25 = 62.5%. Indeed, the same is true if instead of optimistic scenarios (the increases in relation to the base), one analyses the drop in *EBIT* relative to the base (pessimistic scenarios). If, say, a 10% drop in *EBIT* is studied, *EAT* drops to 56.7, hence again by 12.5%. A more dramatic scenario of a 50% slump in *EBIT* translates into a 62.5% slump in *EAT* – always ¼ more than the *EBIT* change itself. Table 1 summarizes various scenarios for $EBIT_1$ illustrating that ***DFL* remains constant regardless of the size of the analysed change in operating profitability.** This is so as its value, for any given level of interest charged, depends solely on the level of the base.

Let us now change, without changing analysed scenarios, the value of the base 100 first to 80, referred to as the low base $EBIT_L < EBIT_0$ and subsequently to 120, referred to as the high base $EBIT_H > EBIT_0$. The scenarios, identical to those in Table 1, span from a loss of 40 to a profit of 300. Tables 2 and 3 illustrate the changes in *EAT* for the low and the high base respectively.

Table 1. *DFL* vs. different scenarios

$EBIT_0$	Int	EBT_0	EAT_0	$EBIT_1$	EAT_1	$\%\Delta EBIT$	$\%\Delta EAT$	DFL
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		(1)-(2)	(3)×(1-0.19%)		[(5)-(2)]×(1-19%)	(5)/(1)-1	(6)/(4)-1	(8)/(7)
100.0	20.0	80.0	64.8	**−40.0**	−48.6	−140.0	−175.0	1.25
100.0	20.0	80.0	64.8	**−20.0**	−32.4	−120.0	−150.0	1.25
100.0	20.0	80.0	64.8	**20.0**	0.0	−80.0	−100.0	1.25
100.0	20.0	80.0	64.8	**50.0**	24.3	−50.0	−62.5	1.25
100.0	20.0	80.0	64.8	**90.0**	56.7	−10.0	−12.5	1.25
100.0	20.0	80.0	64.8	**100.0**	64.8	0.0	0.0	Na
100.0	20.0	80.0	64.8	**120.0**	81.0	20.0	25.0	1.25
100.0	20.0	80.0	64.8	**150.0**	105.3	50.0	62.5	1.25
100.0	20.0	80.0	64.8	**200.0**	145.8	100.0	125.0	1.25
100.0	20.0	80.0	64.8	**300.0**	226.8	200.0	250.0	1.25

Source: own work.

Table 2. Low base *DFL* vs. different scenarios

$EBIT_L$	Int	EBT_L	EAT_L	$EBIT_1$	EAT_1	$\%\Delta EBIT$	$\%\Delta EAT$	DFL
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		(1)-(2)	(3)×(1-0.19%)		[(5)-(2)]×(1-19%)	(5)/(1)-1	(6)/(4)-1	(8)/(7)
80.0	20.0	60.0	48.6	**−40.0**	−48.6	−150.0	−200.0	1.33
80.0	20.0	60.0	48.6	**−20.0**	−32.4	−125.0	−166.7	1.33
80.0	20.0	60.0	48.6	**20.0**	0.0	−75.0	−100.0	1.33
80.0	20.0	60.0	48.6	**50.0**	24.3	−37.5	−50.0	1.33
80.0	20.0	60.0	48.6	**90.0**	56.7	12.5	16.7	1.33
80.0	20.0	60.0	48.6	**100.0**	64.8	25.0	33.3	1.33
80.0	20.0	60.0	48.6	**120.0**	81.0	50.0	66.7	1.33
80.0	20.0	60.0	48.6	**150.0**	105.3	87.5	116.7	1.33
80.0	20.0	60.0	48.6	**200.0**	145.8	150.0	200.0	1.33
80.0	20.0	60.0	48.6	**300.0**	226.8	275.0	366.7	1.33

Source: own work.

With the base of 80 any change in *EBIT* gets levered up by ⅓ as *DFL*=1.33. For example 25% growth in *EBIT* translates into a 33% increase in *EAT*, while a 37.5% drop in *EBIT* results in 50.0% drop in *EAT* (see Table 2). With the base of 120, any change in *EBIT* gets levered up by only 1/5 as *DFL* = 1.20. For example, 25% growth in *EBIT* translates into a mere 30% increase in *EAT* against the increase of 33% for the low base, while a drop of 25% results in a 30% drop in *EAT*.

Table 3. High base *DFL* vs. different scenarios

$EBIT_H$	Int	EBT_H	EAT_H	$EBIT_1$	EAT_1	%ΔEBIT	%ΔEAT	DFL
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		(1)-(2)	(3)×(1-0.19%)		[(5)-(2)]×(1-19%)	(5)/(1)-1	(6)/(4)-1	(8)/(7)
120.0	20.0	100.0	81.0	**−40.0**	−48.6	−133.3	−160.0	1.20
120.0	20.0	100.0	81.0	**−20.0**	−32.4	−116.7	−140.0	1.20
120.0	20.0	100.0	81.0	**20.0**	0.0	−83.3	−100.0	1.20
120.0	20.0	100.0	81.0	**50.0**	24.3	−58.3	−70.0	1.20
120.0	20.0	100.0	81.0	**90.0**	56.7	−25.0	−30.0	1.20
120.0	20.0	100.0	81.0	**100.0**	64.8	−16.7	−20.0	1.20
120.0	20.0	100.0	81.0	**120.0**	81.0	0.0	0.0	Na
120.0	20.0	100.0	81.0	**150.0**	105.3	25.0	30.0	1.20
120.0	20.0	100.0	81.0	**200.0**	145.8	66.7	80.0	1.20
120.0	20.0	100.0	81.0	**300.0**	226.8	150.0	180.0	1.20

Source: own work.

Table 4. Different base *DFL* vs. 10% growth in *EBIT*

$EBIT_0$	Int	EBT_0	EAT_0	$EBIT_1$	EAT_1	%ΔEBIT	%ΔEAT	DFL
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		(1)-(2)	(3)×(1-0.19%)		[(5)-(2)]×(1-19%)	(5)/(1)-1	(6)/(4)-1	(8)/(7)
21.0	20.0	1.0	0.8	**23.1**	2.5	10.0	210.0	21.00
22.0	20.0	2.0	1.6	**24.2**	3.4	10.0	110.0	11.00
23.0	20.0	3.0	2.4	**25.3**	4.3	10.0	76.7	7.67
24.0	20.0	4.0	3.2	**26.4**	5.2	10.0	60.0	6.00
25.0	20.0	5.0	4.1	**27.5**	6.1	10.0	50.0	5.00
30.0	20.0	10.0	8.1	**33.0**	10.5	10.0	30.0	3.00
40.0	20.0	20.0	16.2	**44.0**	19.4	10.0	20.0	2.00
50.0	20.0	30.0	24.3	**55.0**	28.4	10.0	16.7	1.67
60.0	20.0	40.0	32.4	**66.0**	37.3	10.0	15.0	1.50
70.0	20.0	50.0	40.5	**77.0**	46.2	10.0	14.0	1.40
80.0	20.0	60.0	48.6	**88.0**	55.1	10.0	13.3	1.33
90.0	20.0	70.0	56.7	**99.0**	64.0	10.0	12.9	1.29
100.0	20.0	80.0	64.8	**110.0**	72.9	10.0	12.5	1.25
120.0	20.0	100.0	81.0	**132.0**	90.7	10.0	12.0	1.20
150.0	20.0	130.0	105.3	**165.0**	117.5	10.0	11.5	1.15
200.0	20.0	180.0	145.8	**220.0**	162.0	10.0	11.1	1.11
300.0	20.0	280.0	226.8	**330.0**	251.1	10.0	10.7	1.07
500.0	20.0	480.0	388.8	**550.0**	429.3	10.0	10.4	1.04
1000.0	20.0	980.0	793.8	**1100.0**	874.8	10.0	10.2	1.02

Source: own work.

Table 4 illustrates the way *DFL* changes as a result of the change in the base. As the value of *DFL* does not depend on the size of the distance between the scenario and the base, table 4 describes only one positive scenario of $EBIT_1$, i.e. 10% growth against the base. The value of *DFL* ranges from *DFL*=1.00 for $EBIT_0 = +\infty$ to $DFL = +\infty$ for $EBIT_0 = Int$ (see Fig. 1). The lower base, the higher *DFL*, i.e. the leverage effect of a 10% increase in *EBIT* results in a greater percentage change in *EAT* for the lower base. For example, for the base of 21, the 10% change in *EBIT* results in a 210% increase in *EAT*, while the same 10% growth in *EBIT* when the base of 40 is used results merely in a 20% change in *EAT*. The large base of 300, 500, or 1000 results in only marginally higher growth in *EAT* compared to the growth in *EBIT*.

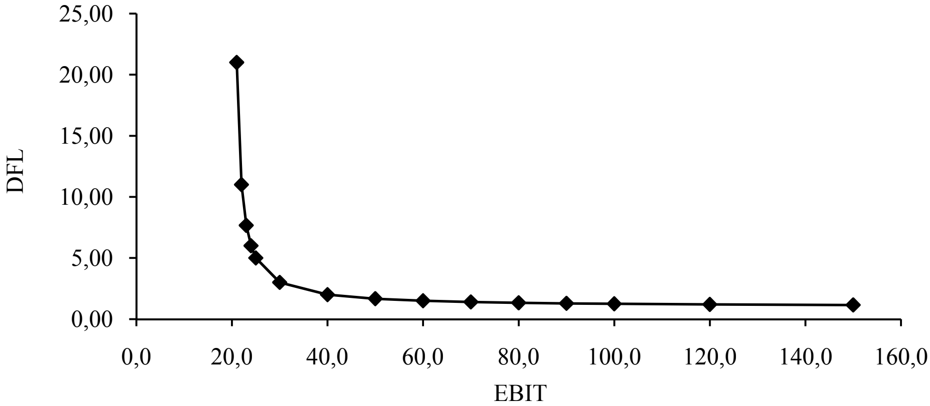

Fig. 1. *DFL* for different values of the base

Source: own work.

Table 5 shows how one fixed scenario can be viewed differently from the perspective of different bases. The table illustrates the impact of the change in *EBIT* on *EAT* for different values of the base assuming nominal scenario is fixed at $EBIT_1 = 150$. This scenario is an optimistic scenario for some bases, where $EBIT_0 < 150$, and a pessimistic scenario for others, where $EBIT_0 > 150$. As the value of $EBIT_1$ does not have an impact on *DFL*, the values of *DFL* in Table 5 are identical to those already produced in Table 4.

Table 5 shows that the value of EAT_1 depends, at a given level of financial interest charged, solely on the value of $EBIT_1$. Not surprisingly it does not depend on the base and consequently on *DFL*, or to put it differently, the level of EAT_1 can be reproduced within a framework of any base and any *DFL* whatsoever. For example: $EBIT_1 = 150$ implies a dramatic increase of 87.5% relative to the low base of $EBIT_L = 80$, yet only moderate growth of 25% for the high base of $EBIT_H = 120$. Those growth rates of *EBIT* are subsequently levered up by *DFL* to produce growth rates

Table 5. Different base *DFL* vs. a fixed scenario of $EBIT_1 = 150$

$EBIT_0$	Int	EBT_0	EAT_0	$EBIT_1$	EAT_1	$\%\Delta EBIT$	$\%\Delta EAT$	DFL
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		(1)-(2)	(3)×(1-0.19%)		[(5)-(2)]×(1-19%)	(5)/(1)-1	(6)/(4)-1	(8)/(7)
21.0	20.0	1.0	0.8	**150.0**	105.3	614.3	12900.0	21.00
22.0	20.0	2.0	1.6	**150.0**	105.3	581.8	6400.0	11.00
23.0	20.0	3.0	2.4	**150.0**	105.3	552.2	4233.3	7.67
24.0	20.0	4.0	3.2	**150.0**	105.3	525.0	3150.0	6.00
25.0	20.0	5.0	4.1	**150.0**	105.3	500.0	2500.0	5.00
30.0	20.0	10.0	8.1	**150.0**	105.3	400.0	1200.0	3.00
40.0	20.0	20.0	16.2	**150.0**	105.3	275.0	550.0	2.00
50.0	20.0	30.0	24.3	**150.0**	105.3	200.0	333.3	1.67
60.0	20.0	40.0	32.4	**150.0**	105.3	150.0	225.0	1.50
70.0	20.0	50.0	40.5	**150.0**	105.3	114.3	160.0	1.40
80.0	20.0	60.0	48.6	**150.0**	105.3	87.5	116.7	1.33
90.0	20.0	70.0	56.7	**150.0**	105.3	66.7	85.7	1.29
100.0	20.0	80.0	64.8	**150.0**	105.3	50.0	62.5	1.25
120.0	20.0	100.0	81.0	**150.0**	105.3	25.0	30.0	1.20
150.0	20.0	130.0	105.3	**150.0**	105.3	0.0	0.0	Na
200.0	20.0	180.0	145.8	**150.0**	105.3	–25.0	–27.8	1.11
300.0	20.0	280.0	226.8	**150.0**	105.3	–50.0	–53.6	1.07
500.0	20.0	480.0	388.8	**150.0**	105.3	–70.0	–72.9	1.04
1000.0	20.0	980.0	793.8	**150.0**	105.3	–85.0	–86.7	1.02

Source: own work.

for *EAT*. For the low base *DFL* = 1.33, while for the high base *DFL* = 1.25. Consequently the changes in *EAT* are 116.7% = 87.5% × 1.33 for the low base and 30% = 25% × 1.25 for the high base. The difference may look impressive but this is because the change in the base *EBIT* is followed by the change in the base *EAT*. When growth of 116.7% is applied to the low base EAT_L of 48.6 we end up with an identical reading of 105.3 as in the case when growth of 30% is applied to the high base EAT_H of 81 (see Table 6).

Indeed, any value of *DFL* could be applied with no impact on the final outcome. Equation (2) proves this point algebraically.[2]

$$EAT_1 = EAT_0 \times (1 + DFL_0 \times \frac{EBIT_1 - EBIT_0}{EBIT_0}) = EAT_0 \times (1 + DFL_0 \times \%\Delta EBIT) \quad (2)$$

[2] $EAT_1 = (1-T) \times (EBIT_1 - Int) = (1-T) \times [EBIT_1 - (EBIT_0 - EBT_0)]$

$= (1-T) \times (EBT_0 + EBIT_0 \times \frac{EBIT_1 - EBIT_0}{EBIT_0}) = EAT_0 \times (1 + DFL_0 \times \%\Delta EBIT)$

Table 6. Low and high base *DFL* vs. a scenario of $EBIT_1 = 150$

		L Low base	H High base
(1)	Base *EBIT*	80.0	120.0
(2)	*Int*	20.0	20.0
(3)=(1)–(2)	Base *EBT*	60.0	100.0
(4)=(1–19%)×(3)	Base *EAT*	48.6	81.0
(5)=(1)/(3)	***DFL***	**1.33**	**1.20**
(6)	$EBIT_1$	150.0	150.0
(7)=(6)/(1)-1	**$\%\Delta EBIT$**	**87.5%**	**25.0%**
(8)=(7)×(5)	**$\%\Delta EAT$**	**116.7%**	**33.3%**
(9)=(4)×(1+(8))	EAT_1	105.3	105.3

Source: own work.

There is no particular condition $EBIT_0$ is to meet for the equation (2) to be true.

Depending on whether the low base $EBIT_L$ or the high base $EBIT_H$ are used, EAT_1 is produced either as:

$$EAT_1 = EAT_L \times (1 + DFL_L \times \frac{EBIT_1 - EBIT_L}{EBIT_L}) \tag{2a}$$

or as:

$$EAT_1 = EAT_H \times (1 + DFL_H \times \frac{EBIT_1 - EBIT_H}{EBIT_H}) \tag{2b}$$

The bases can easily be switched from low to high and the other way round without affecting the conclusions regarding *EAT*. Below the algorithm describing how to translate the changes from the perspective of the low base to the perspective of the high base is presented :

$$\begin{aligned} EAT_1 = EAT_L \times (1 + DFL_L \times \frac{EBIT_H - EBIT_L}{EBIT_L}) \times \\ \times (1 + DFL_H \times \frac{EBIT_1 - EBIT_H}{EBIT_H}) \end{aligned} \tag{3}$$

The first part of the right hand side of (3) is the low base EAT_L, which subsequently gets levered up by the low base DFL_L to produce the high base EAT_H (the product of the first two factors on the right hand side of 3) which in turn gets levered up by the high base *DFL* to produce EAT_1.

Using our example of $EBIT_L = 80$ and $EBIT_H = 120$ and scenario of $EBIT_1 = 150$ (see Table 7), we can, first using the low base, discover the scale of the percentage

Table 7. The switch between different *DFL* levels vs. a scenario of $EBIT_1 = 150$

		From low to high base		From high to low base	
		L Low base	H High base	H High base	L Low base
(1)	Base *EBIT*	80.0	120.0	120.0	80.0
(2)	*Int*	20.0	20.0	20.0	20.0
(3)=(1)–(2)	Base *EBT*	60.0	100.0	100.0	60.0
(4)=(1–19%)×(3)	Base *EAT*	48.6	81.0	81.0	48.6
(5)=(1)/(3)	DFL	1.33	1.20	1.20	1.33
(6)	$EBIT_1$	120.0	150.0	80.0	150.0
(7)=(6)/(1)-1	%Δ*EBIT*	50.0%	25.0%	–33.0%	87.5%
(8)=(7)×(5)	%Δ*EAT*	66.7%	33.3%	–40.0%	116.7%
(9)=(4)×(1+(8))	EAT_1	81.0	105.3	48.6	105.3

Source: own work.

change in *EAT* resulting from the switch from the low to the high base. $EBIT_H = 120$ implies a 50% change in *EBIT* which translates into 66.7% change in *EAT* using the low base *DFL* of 1.33. This results in *EAT* to equal 81.0=48.6×(1+66.7%), i.e. the high base value of *EAT*. What follows from this point onwards is the high base analysis using $EBIT_1$ of 150 and the high base profit figures of $EBIT_H = 120$ and $EAT_H = 81.0$. Similarly, the transformation from the high to the low base analysis can be performed (see Table 7).

3. Does higher *DFL* imply higher risk?

The higher value of *DFL* is viewed by some as the indication of the increase in financial risk faced by the company. This claim is reviewed now in this section in more detail. It is true that the scenarios seem more risky when viewed from the perspective of the lower base (the scenarios presented in the form of the percentage difference to the base will be referred from now on as relative scenarios). Indeed, what is a 25% change for $EBIT_1 = 150$ with the high base of 120 is a 87.5% change for the low base of 80. Similarly, 66.7% and 150% relative scenarios with the high base become 150% and 275% respectively with the low base (see tables 2-3). In addition, pessimistic scenarios seem even more pessimistic when presented in the relative format: the change of –116.7% and –133.3% with the high base are –125% and –150% respectively with the low one (Tables 2 and 3).

However, this explanation does not hold for all pessimistic scenarios. For those pessimistic scenarios with respect to the high base which are profitable, i.e. $0 < EBIT_1 < EBIT_H$, there is no indication of higher risk in the sense mentioned above at all when the low base is applied. Quite the opposite, those scenarios seem less risky when $0 < EBIT_1 < EBIT_L$ (–58.3% and –83.3% in Table 3 are only –37.5% and –75% respectively in Table 2) or even turn into the optimistic scenarios, when $EBIT_L < EBIT_1 < EBIT_H$. Below the summary of the changes in the perception of the analysed scenarios after the decrease in the base is presented:

– Optimistic scenarios in the sense that $EBIT_1 > EBIT_H > 0$ remain optimistic after the drop in the base ($EBIT_1 > EBIT_L > 0$); actually they seem even more optimistic as relative scenarios as $EBIT_1/EBIT_L > EBIT_1/EBIT_H$.
– Some scenarios, which are pessimistic against the high base become optimistic with the low base when $EBIT_L < EBIT_1 < EBIT_H$.
– Those pessimistic scenarios, which remain pessimistic even when the lower base is applied ($EBIT_1 < EBIT_L$) seem to be less pessimistic in the relative format for the scenarios which are in the black, i.e. $EBIT_1 > 0$ and more pessimistic for those scenarios which are in the red, i.e. $EBIT_1 < 0$.

It is not therefore true that all relative scenarios become more (less) risky in the way defined above after the introduction of the lower (higher) base. Those who are advocates of *DFL* must find therefore other ways to argue that the lower base-higher *DFL* framework means higher risk. One way to follow is presented below.

Let l and h be relative scenarios generated from a nominal scenario $EBIT_1$ with the low $EBIT_L$ and the high $EBIT_H$ base respectively:

$$l = \frac{EBIT_1}{EBIT_L} - 1 \tag{4}$$

$$h = \frac{EBIT_1}{EBIT_H} - 1 \tag{5}$$

Then it follows that the relationship between the relative scenarios can be presented as:

$$l = \frac{EBIT_H - EBIT_L}{EBIT_L} + \frac{EBIT_H}{EBIT_L} \times h \tag{6}$$

or

$$l = -100\% + \frac{EBIT_H}{EBIT_L} \times (h - 100\%) \tag{7}$$

This in turn leads to the conclusion that the variance of the low base relative scenarios is higher than that for the high base, or:

$$stdev(l) = \frac{EBIT_H}{EBIT_L} \times stdev(h) > stdev(h) \tag{8}$$

The increase in the standard deviation of the relative scenarios after the drop in the base might be argued to present a legitimate rationale for the claim that the risk does indeed increase in the low base-high *DFL* case.

Another argument for this claim is provided by a modified version of a cumulative distribution function *CDF* of the relative scenarios. Let $F(x)$ be a cumulative distribution function, where x is a relative scenario, i.e. $F(x)$ describes the probability of registering scenario which is x percent or less away from the base. Let folded cumulative distribution function $G(x)$ be identical to $F(x)$ for all values of $x < -100\%$ and equal to $[1- F(x)]$ for $x > -100\%$. Then $G(x)$ has the following features:

– For the upslope, i.e. for extremely pessimistic scenarios where $x < -100\%$, $G(x)$ determines the probability of getting *EBIT* which in relative form is lower than or equal to the base by more than x%, and for the downslope, i.e. when $x > -100\%$, $G(x)$ determines the probability of reaching *EBIT* which in relative form is greater than or equal to x%.
– $G(x)$ is discontinuous at $x = -100\%$, unless median of the relative scenarios distribution happens to fall at exactly $x = -100\%$.[3]

$G(x)$ for the low base is always greater than $G(x)$ for the high base for both down as well as upslope. This, some could argue, is precisely what leverage and increased risk actually is. In Fig. 2 this can be seen in thicker tails of folded *CDF* for the low base. Should still lower base be selected, the arms of the folded cumulative function would go even higher.

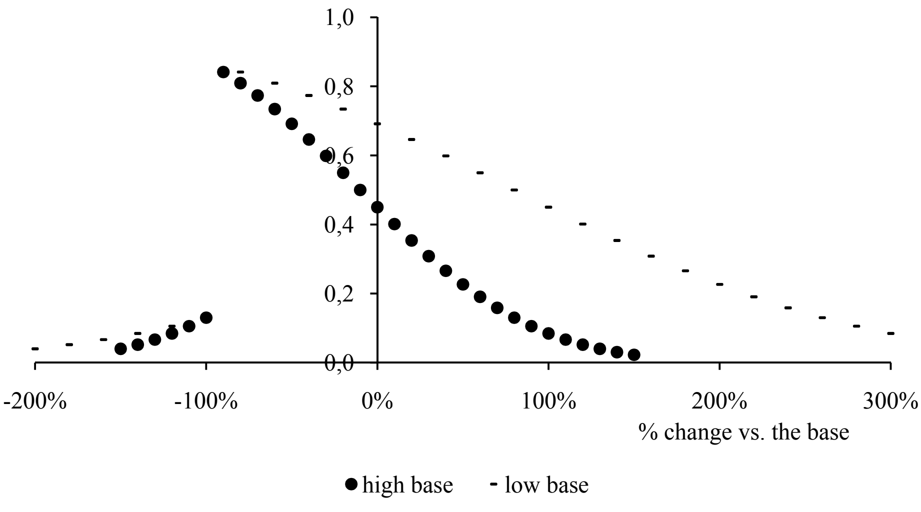

Fig. 2. Folded cumulative distribution function for relative scenarios

Source: own work.

[3] The special role of $x = -100\%$ played in the construction of a folded cumulative distribution function comes from the fact that at $EBIT_1 = 0$, all relative scenarios are –100% regardless of the level of the base (see equation (7)). For more on folded cumulative distribution functions used as a tool to describe leverage situations see [Berent 2010a].

The advocates of the claim that higher *DFL* means higher risk may therefore use the folded cumulative function of relative scenarios and/or the higher standard deviation of relative scenarios with the low base as evidence supporting their claim. They may further argue that even if the decrease (increase) in the base is not accompanied by any change in actual scenarios, the risk gets higher (lower) because those seemingly unaltered scenarios in nominal terms tend to account for more (less) of the base. This stance may seem appealing but has severe drawbacks described below.

It is universally agreed that the risk of any economic venture implies some kind of uncertainty associated with the unknown outcomes in the future. Theory of finance and asset pricing models in particular link the (undiversifiable) risk to the return on capital employed, not to relative rates calculated against some base, chosen – to make it worse – in an arbitrary fashion. Hence, if we assume the change in the base does not affect nominal scenarios studied and it certainly does not affect capital employed then such a change cannot affect returns on capital either. In the light of the modern theory of finance therefore, there is little room for claims that mere changes in the base in *DFL*-type analysis affect risk-return profile of the project/company and hence its cost of capital and valuation.

To illustrate inadequacy of *DFL* approach to risk measurement, one can imagine two ventures A and B: venture A is characterized by large variability of potential future outcomes, venture B is more like a risk-free asset, showing marginally low level of uncertainty. However, if one chooses to use an extremely high base for the risky venture A, one will not spot high risk using *DFL* at all. The high base results in a low *DFL* and consequently any change in operating profits relative to this base is not levered up much and consequently does not result in a big relative change in net profits. If instead the low base is applied to study the low risk venture B, the analyst is bound to conclude that any future change relative to the base results in far more than proportional change in net profit. As a result of using *DFL*, one can wrongly conclude that risky venture A is not risky at all, while the nearly risk-free venture B is very risky.

This example illustrates two important drawbacks of *DFL*: its ignorance of how far the base is from the distribution of potential scenarios and its inability to see the true variability of potential outcomes. Selecting the base miles away from likely outcomes tells us less about the risk of a venture (seemingly low) and much more about the ignorance of the analyst. On the other hand, the answer to the question: what happens with *EAT*, if *EBIT* changes by 1, 10, or 50% says nothing about how likely these changes are. All in all, we must conclude that the "risk" spotted by detection of high *DFL* has little to do with the way modern theory of finance understands risk and more with the arbitrary choice of the benchmark used.[4]

[4] In his Nobel Memorial Prize Lecture presented to the Royal Swedish Academy of Sciences, when referring to higher variability in returns and the concept of risk associated with this variability, Miller writes: "And this greater variability of prospective rates of return to leveraged shareholders means greater risk, in precisely the sense used by my colleagues, Harry Markowitz and William Sharpe"

4. *DFL* as a language convention and a managerial tool

This is not to say that the "risk" observed after the decrease in the base is completely irrelevant. As was shown above, using *DFL* generates mathematically correct answers to the question: what happens to net profit should operating profit change by *x*%, whatever the base is used. The previous analysis proved also that there were countless ways to explain how any given $EBIT_1$ translates into EAT_1 using different bases. Indeed, it looks like *DFL* provides nothing but a language, which explains the way $EBIT_1$ translates into net profit with the help of *DFL*. Different *DFLs* offer different languages. The change in the base implies a different reference number thus supplying us with different linguistic rules to describe the scenario analysed. We conclude therefore that *DFL* is not a measure of risk per se but a method of communicating (potential) results with the help of different benchmarks.

In Table 5, we present just few ways one can describe how the scenario of $EBIT_1 = 150$ transforms itself into $EAT_1 = 105.3$ with the help of various *DFLs*. In Table 8 below, we use data from Table 5 with its columns reshuffled and with the *DFL* interpretation provided.

It is clear from Table 8 that all comments are correct. All of them therefore are different ways (languages) to communicate exactly the same message. Any *DFL* is mathematically equally successful. Does this mean that all *EBIT* bases/benchmarks are equally relevant to the business practice? The answer to this question is simple: the more meaningful the base to managers, the more meaningful *DFL* analysis, and the more clear language it provides. One can easily imagine legitimate candidates for meaningful bases to be e.g.:

– last year earnings;
– expected value of earnings (in statistical terms);
– earnings required by owners;
– lowest acceptable level of earnings;
– record high level of earnings etc.

Let us assume that (see Table 8):

– the company's *EBIT* last year was 24.0;
– the lowest acceptable level of *EBIT* for the current year is 50.0;
– the required by shareholders level of *EBIT* for the current year is 120;
– the expected value for the current year *EBIT* is 300;
– the highest in the company's history level of *EBIT* is 1000.

Consequently, all the following statements are not only mathematically true but also meaningful to practitioners:

[Miller 1991, p. 108]. By referring to Markowitz' and Sharpe's understanding of risk, Miller refers implicitly to variance and beta of returns respectively. Ironically, in the numerical example he uses in the lecture, he does not calculate any of the two. Instead he computes *DFL*, which he wrongly assumes to be a good proxy for the risk (more on Miller's controversial statement, see [Berent 2010b]).

– if the company generates *EBIT* in the current year which is 1% higher than that recorded last year, i.e. *EBIT* = 24, than its net profit will be 6% higher than that recorded last year, i.e. *EAT* = 3.2;
– if the company generates *EBIT* in the current year which is 1% higher than the lowest acceptable level of *EBIT* = 50, than its net profit will be 1.67% higher than the lowest acceptable level of *EAT* = 24.3;
– if the company generates *EBIT* in the current year which is 1% higher than the required by investors level of *EBIT* = 120, than its net profit will be 1.2% higher than the *EAT* = 81 required by investors;
– if the company generates *EBIT* in the current year which is 1% higher than the mean (expected) value of *EBIT* = 300, than its net profit will be 1.07% higher than the mean (expected) value of *EAT* = 226.8;
– if the company generates *EBIT* in the current year which is 1% higher than the record high level of *EBIT* = 1000, than its net profit will be 1.02% higher than the record high level of *EAT* = 793.8.

The above statements can be reproduced for a fixed scenario, say $EBIT_1$ = 150, rather than for a fixed relative scenario of 1%. Little changes: *DFL* is different for different bases, the language used is meaningful for those who use it, yet nothing is

Table 8. *DFL* as a language convention

$EBIT_0$	$EBIT_1$	%ΔEBIT	DFL	%ΔEAT	EAT_0	EAT_1	Comment
(1)	(2)	(3)=(2)/(1)-1	(4)	(5)=(3)×(4)	(6)	(7)=(6)××(1+(5))	
24.0	**150.0**	525.0	6.00	3150.0	3.2	105.3	$EBIT_1$ is 525% higher than the base; as *DFL*=6, net profit is 3150% higher than EAT_1
50.0	**150.0**	200.0	1.67	333.3	24.3	105.3	$EBIT_1$ is 200% higher than the base; as *DFL*=1.67, net profit is 333.3% higher than EAT_1
120.0	**150.0**	25.0	1.20	30.0	81.0	105.3	$EBIT_1$ is only 25% higher than the base; as *DFL* = 1.2 net profit is 30% higher than EAT_1
300.0	**150.0**	–50.0	1.07	–53.6	226.8	105.3	$EBIT_1$ is 50% lower than the base; as *DFL* = 1.04 net profit changes at a similar rate (–53.6%)
1000.0	**150.0**	–85.0	1.02	–86.7	793.8	105.3	$EBIT_1$ is 85% lower than the base; as *DFL* = 1.02, the percentage change in EAT_1 is almost identical (–86.7%)

Source: own work.

learnt about the risk of the venture. The nature of *DFL* as a linguistic convention can further be seen if $EBIT_1$ = 150 is assumed to be risk free. As no risk is present, the comments from Table 8 – still mathematically correct yet void of any practical meaning – show simply different ways of presenting this risk-free scenario. They certainly do not point to any risk which, in this case, is simply absent.

The users of *DFL* have a lot of discretion in choosing between various levels of the base. This freedom is only limited by one condition that $EBIT_0 > Int$, the condition which secures intuitive interpretation of *DFL* as a leverage measure. For $EBIT_0 < Int$, *DFL* may be negative or a fraction and hence cannot be interpreted as a leverage ratio. Needless to say, mathematically everything remains correct regardless of whether if *DFL* is greater than one or not.

Also, a word of caution is necessary for those who use *DFL* with historic results as a base (see [Michalski 2010, pp. 44-45]). If last year or record high historic results are used in the way they are used above, one has to make some additional assumptions of no change in interest charged *Int* and no change in tax rate for the meaningful use of *DFL*. As Table 9 illustrates, any change either in interest paid, through the interest rate change or the change in the debt load, or/and in effective tax rate results in the growth in net profit different from that projected by *DFL* calculation.

Table 9. *DFL* with historic bases

		No change in interest No change in tax rate		Change in interest No change in tax rate		No change in interest Change in tax rate	
	$EBIT_0$	$EBIT_1$	%Δ vs base		%Δ vs base		%Δ vs base
EBIT	100.0	120.0	20.0%	120.0	20.0%	120.0	20.0%
Int	20.0	20.0	0.0%	10.0	–50.0%	20.0	0.0%
EBT	80.0	100.0	25.0%	110.0	37.5%	100.0	25.0%
Tax rate	19%	19%	0.0%	19%	0.0%	30%	57.9%
EAT	64.8	81.0	25.0%	89.1	37.5%	70.0	8.0%
DFL	1.25		25%=1.25×20%		37.5%≠1.25×20%		8%≠1.25×20%

Source: own work.

5. Conclusions

With the help of *DFL* one can easily calculate the value of (future) net profit, which results from the change in operating income. Although any value of *DFL* is mathematically acceptable (set A in Fig. 3), only those greater than one can be interpreted as leverage ratios (set B).

In such cases, *DFL* informs about the percentage change in *EAT* which results from a 1% change in *EBIT*. The value of *DFL* is determined by the value of the base: the higher (lower) the profit levels used as the base, assuming interest paid is fixed,

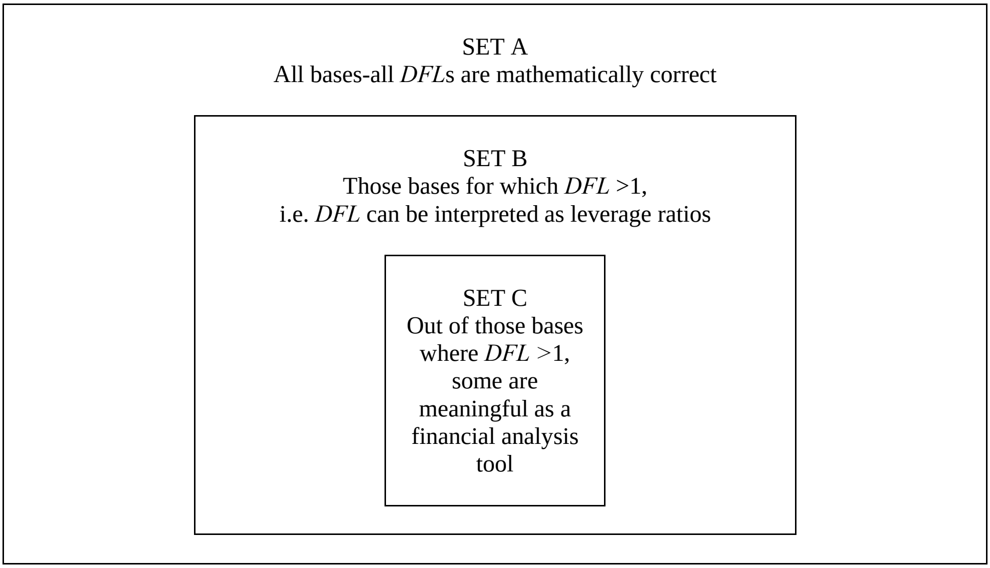

Fig. 3. Application of different bases in *DFL* calculation

Source: own work.

the lower (higher) the level of *DFL*. As shown above, one can easily swap from one base to the other to reproduce the same profit scenario. It can therefore be argued that *DFL* provides only a language convention in which an analysed scenario is described. This in turn means that *DFL* has little to do with the risk measurement as understood by the modern investment and finance theory. Being only a language, some *DFL* are more meaningful than others. If the managers or academics use e.g. past historic profits or forecasted expected levels, one can argue that *DFL* provides a useful managerial tool, or base, to talk about (future) scenarios. It follows that only some *DFL* values are meaningful for practitioners (Set C). One should however remember that the degree of discretion in choosing the base for *DFL* calculation is massive, hence it should be used with utmost caution.

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BAZA W OBLICZANIU *DFL*

Streszczenie: Jednym z najbardziej popularnych mierników dźwigni finansowej jest wskaźnik stopnia dźwigni finansowej *DFL* (*Degree of Financial Leverage*). Wskaźnik ten, jako miernik elastyczności, informuje o skali relatywnej (procentowej) zmiany zysku netto wywołanej zmianą zysku operacyjnego o 1%. Niniejszy artykuł podejmuje temat wartości bazowej zysku operacyjnego i zysku netto, które służą do obliczania *DFL*. Jako że zmiana wartości bazowych zysku zmienia poziom *DFL*, problem wyboru bazy jest niezwykle istotny. W artykule przedstawiono argumenty za traktowaniem *DFL* jako konwencji językowej, która niekiedy może pełnić użyteczna rolę w analizie finansowej. Ze względu na umowność obecną przy wyborze bazy, koniecznej do obliczania *DFL*, *w*skaźnika tego nie należy traktować jako wskaźnika ryzyka finansowego w pojęciu nowoczesnej teorii finansów.