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INVESTIGATION ABOUT THE CONTRIBUTION OF TECTONIC CONDITIONS TO MINING SUBSIDENCE PARAMETERS

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Abstract: Underground infrastructure of any kind can affect the surface by inducing ground movements. The ability of precise subsidence prediction is crucial for environmental management. Prediction methods in practice are mainly based on influence functions that are symmetrical and provide comparably smooth profiles. In the past, deviations from the predictions have been detected. The ability of modern geomonitoring data makes the deviation even more obvious today. One of the reasons for the deviations are the regional tectonic stress conditions. To justify further investigation into the impact of tectonic conditions on the subsidence parameters, numerical experiments were conducted based on a hypothetical case of a homogenous sedimentary rock under different stress conditions. As a result, deviation of up to 7% of the subsidence profile parameters was detected. The results can be considered significant and encourage researchers to investigate the topic further to extend the currently used prediction methods to take into count the tectonic conditions. The research is based on numerical simulation and provides only theoretical result, implementation and validation of the theory in the field are left for further investigation.

Keywords: *subsidence prediction, tectonic conditions, numerical modelling, asymmetry, uplifting*

1. INTRODUCTION

Methods of subsidence prediction are applied to protect objects on the surface or evaluate the potential damage caused by the exploitation of deposits. Although the subsidence

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process is a part of ground deformation, typically its prediction uses a kinematic model geometrically parameterized. Classical prediction methods used in central Europe are based on Knothe's theory (Sroka 1993; 2001) with parameters related to influence functions to adjust the model to the specific case. The choice and popularity of this approach is justified by the limited and interpretable number of input parameters, the ability to determine parameters by geodetic observation and by the suitable accuracy of prediction results. This makes it more applicable in comparison to geomechanical approaches, which require large numbers of parameters that may be not available or have low accuracy.

Historically, the first analytical solution in mine surveying using a stochastic approach to interpret void diffusion was published by Litwiniszyn (1958). Knothe (1953) obtained similar result for statistical interpretation of measured subsidence of a coal mine in Poland. In the field of geomechanics, Berry (1963) compared subsidence with the beam bending process. Both approaches work well within the authors' prescribed conditions that exclude the influence of tectonic conditions.

Subsidence is a complicated process influenced by multiple factors, for example, rock properties, geological structures, groundwater, porosity, stress conditions, etc. The monitoring of subsidence areas under tectonic conditions indicates anomalies in a form of asymmetry, shape anisotropy and uplifting (Awerschin 1947; Sashurin 1999, Busch 2014–2017). Although the origin of the deviations is not properly investigated, different methods have been applied to correct them. Selected aspects of possible reasons for deviations have been investigated: the influence of the Young modulus on the exploitation coefficient by Suchowerska (2016); Xia et al. (2016; 2017) artificially connect stress-realizing activity with changes in the angle of influence. These studies motivate a deeper and wider investigation of deviations and their origins, and as consequence requires understanding of the geomechanical basics of subsidence and current prediction methods. Since the origins of deviations can be different, the task requires foundation and investigation of the factors separately and should be grounded in theory in the context of real cases of deviations.

The referenced publications of Awerschin (1947), Sashurin (1999), Xia et al. (2016–2017), indirectly by Sushwerska (2016) suppose that one of the significant reasons for the deviation can be unaccounted tectonic conditions in other words stress field. Since the limits and magnitudes of the influence of tectonic conditions on mining subsidence have not been covered or documented in literature, this research investigates the topic. It starts with an analysis of the existing methods to define their mathematical limits and leads through an observation of the "anomalies" with the assumption that stress field is a key factor. The investigation is based on the numerical method to exclude the influence of external factors. To build an appropriate experiment the magnitude of applied stress should be realistic therefore a classification of stress regimes is given. The result is presented in the form of profiles under different stress conditions and its parameters, as an influence angle, maximum subsidence, etc.

The aim of the research is to understand the necessity of including tectonic conditions in subsidence prediction methodologies.

2. FOUNDATIONS

2.1. EXISTING PREDICTION METHODS AND ITS LIMITS

This chapter briefly reviews different subsidence prediction methods on the example of 2 different types (empirical and geomechanical) to understand the nature of parameters and qualitative limits of the prediction methods in general. At the end of this chapter, methods are compared and limits discussed.

Geomechanical analytical solution: Berry (1963) provided an analytical solution for elastic behaviour of rock masses limited to a depth up to 70 m. The subsidence at the location $s(x)$ given by:

$$s(x)=\frac{m}{\pi((1-\upsilon_1^2)^{1/2}-\alpha_2)}\left[(1-\upsilon_1^2)^{1/2}\tan^{-1}\frac{2ah_1}{x^2-a^2+h_1^2}-\alpha_2\tan^{-1}\frac{2ah_2}{x^2-a^2+h_2^2}\right], \quad (1)$$

where: a is a length of an excavated part, x – horizontal coordinate relative to the centre of an excavated element, h is a depth, E is Young's modulus, m is a working height, υ is the Poisson's ratio, the deformation of the material in directions perpendicular to the direction of loading. Index 1 of E and υ characteristic means, that the properties are in the plane of isotropy, 2 – that they are normal to the plane of isotropy. $\alpha_{1,2}$, $h_{1,2}$ are define as:

$$\alpha_1=(1-\upsilon_1^2)^{1/2}, \quad (2)$$

$$\alpha_2=(E_1/E_2-\upsilon_2^2)^{1/2}, \quad (3)$$

$$h_{1,2}=\frac{h}{\alpha_{1,2}}. \quad (4)$$

Empirical methods: Litwiniszyn (1958) demonstrated, that a solution for the subsidence description of an incompressible medium obtained at a fixed level has a Gaussian trough. He proved this hypothesis through the physical model of dry sand medium displacement. Similar but more simple version of the solution presented by Knothe (1953) based on coal mine observations in Poland, the solution is accepted by mining companies and different software applications this method (Sroka 2001; Kratzsch 1983; Kwinta, Gradka 2018). Knothe's influence function (Kratzsch 1983):

$$s_K(x)=\frac{S_{\max}}{r_k}\exp\left(\frac{\pi x^2}{r_k^2}\right), \quad (5)$$

where: S_{max} is a maximum final subsidence in the case of critical or supercritical states; $S_{max} = a \cdot m$, where m – thickness of the layer of bed to be exploited, a – an exploitation coefficient, which describes the maximum amount of convergence, which transfers in subsidence on the surface, r_k is a parameter of influence dispersion, or a radius of the main influence range, define as:

$$r_k = H / \mathrm{tg}\,\beta, \tag{6}$$

where: H – a depth of the seam, m, β (γ in some of the other sources) – an angle of the main influence range. Note, the angle of the main influence range is connected with the physical-mechanical properties of the rock mass and rock type.

Other typical subsidence influence functions, e.g., of Bals (1931/32), Konchmański (1955), Bayer, Ruhrkohle–Verfahren and Geertsma (Sroka 1993–2001) are also based on trigonometric function, circle and geomechanical or geometrical parameters. They provide more or less comparable results that are mathematically limited to symmetry and subsidence.

The referenced methods use different input parameters: the geometry-based approaches use the angle of influence γ (e.g., Knothe), the geomechanical approaches use Poisson ratio and/or Young's modulus (e.g., Balls). Since they all provide comparable results, the angle of influence (γ) should relate to geomechanical parameters. The parameters (Young's modulus and Poisson ratio) relate to applied stress, thus γ should also be sensitive to it. Therefore, it is worth investigating cases of subsidence in the field under tectonic conditions to observe if there is an overlap between the higher prediction inaccuracy and tectonic conditions.

2.2. CASES OF SUBSIDENCE UNDER TECTONIC CONDITIONS

The first complete study about the influence of horizontal stress on mine subsidence for real coal deposits was conducted by Awerschin (1947) and was continued for polymetallic deposits by Sashurin (1999). On the basis of ~80 measurements of displacement for 105 subsidence profiles, the presence of unpredictable anomalies was evidenced including:

~25% of the cases is deviation according to vertical displacement (uplift) and

~24% of the cases is deviation regarding horizontal displacement (direction of displacement and value).

Research gave an explicit conclusion: The ground movement parameters are determined not only by the geometrical parameters but also by the initial stress state of the rock mass.

With development of geomonitoring methods such us Photogrammetry, Radar interferometry, GNSS, the ability of measured data of surface deformation is increased, thus the anomalies is becoming more obvious. The comparisons between predicted and measured subsidence, using the InSar and GNSS technique, reveal significant deviation, in particular in the range of influence and asymmetry. In the report from TU Clausthal (Busch 2014–2017), significant uplift has been detected, either the uplifting effect used to be coupled only with a watering influence, which is not the case there. Quasnitza (1985) noticed a shift of subsidence maximum location. Xia et al. (2016) reported a direct relation between stress-relaxing activities and 10 degrees' changes in the influence angle. Vušović et al. (2021) compare a modern stochastic prediction result and real trough of subsidence, however, did not to identify a profile asymmetry, although presents.

To better understand the problem and simulate appropriate results the range of tectonic force magnitude and its representation in an applicable form for a numerical solution should be investigated.

2.3. REVIEW OF STRESS CONDITION FIELDS AND CLASSIFICATIONS

As aforementioned, the locations of the presented cases relate to the regions with tectonic conditions, including faulting regions and inclinations of layers. To describe the stress field region, Anderson (1951) compares horizontal and vertical stresses. His classification describes relative stress magnitudes in the earth's crust based on the magnitude of $\frac{\sigma_{\upsilon}}{\sigma_h}$, for the classification Anderson used $\sigma_{h\ \max}$ as a maximum of horizontal stress, $\sigma_{h\ \min}$ as a minimum of horizontal stress and σ_{υ} as a vertical stress. Anderson's classification and relative locations according to Okal (2009), Tsanakas et al. (2019), Aurelio (2008), Gledhill et al. (2000) and Shikakura et al. (2014) presented as:

- Normal faulting region identified by $\sigma_{\upsilon} > \sigma_{h\ \max}$ ($\sigma_{h\ \max} \geq 0.6\sigma_{\upsilon}$, for example, Central Greece, Illinois, Indiana, United States, Pennines);
- Strike-slip faulting region identified by ($\sigma_{h\ \max} > \sigma_{\upsilon} > \sigma_{h\ \min}$ ($\sigma_h = 0.6{\sim}1.6\sigma_{\upsilon}$, for example, Philippines; Nova Scotia; Atacama Desert, Chile; Turkey);
- Reverse faulting region identified by $\sigma_{h\ \min} > \sigma_{\upsilon}$ ($\sigma_{h\ \min} \geq 1.6{\sim}2.3\sigma_{\upsilon}$, for example, South Island, New Zealand, Kinki region Japan, Alps).

These stress regimes are typical for the following regions: Western Europe, including the coalfields of Germany and the UK; some of the coalfields in Australia; Eastern US "mid-plate" region and the Western US (Wang et al. 2012). Moreover, the ratio $\frac{\sigma_h}{\sigma_{\upsilon}}$ can widely vary In-situ, for example from 2.75 (335 m depth) to 1.27 (495 m depth) (Lekhnitskii 1963). Since in most cases vertical stress can be presented as the weight of overlying rocks, the influence factor should rather be connected with changes in applied horizontal stress. The exemptions, when vertical stress cannot be defined by the weight of overlying rocks, are excluded from the study, although since the valuation of the stress taken into account in the form of stress ratio, the question is partially covered.

2.4. CONCLUSIONS BASED ON LITERATURE OVERVIEW

The combination of the current prediction methods limitations and discovered cases of “anomalies” leads to the following suppose: One of the high potential directions to improve the subsidence prediction method is exceeding the aforementioned limitations. In the article is discovering if the reason for “anomalies” is tectonic conditions; which characteristics of the subsidence profile it influences and if the influence is significant. Based on the literature overview, the following hypotheses are suggested:

- Horizontal stress influences the angle of subsidence influence and exploitation coefficient;
- Horizontal stress could be a reason for uplift, although incomparable with subsidence magnitude.

The hypothesis that tectonic force is a reason for an asymmetric subsidence is left for further investigation.

3. EXPEREMENT DESIGN

3.1. METHODOLOGY

This section presents the methodology used to investigate the influence of horizontal stress, its justification, used data and parameters. To investigate the hypothesis, 5 profile of subsidence has been numerical simulated in geotechnics and analysed relative to each other. The chain of the process is presented in Fig. 1.

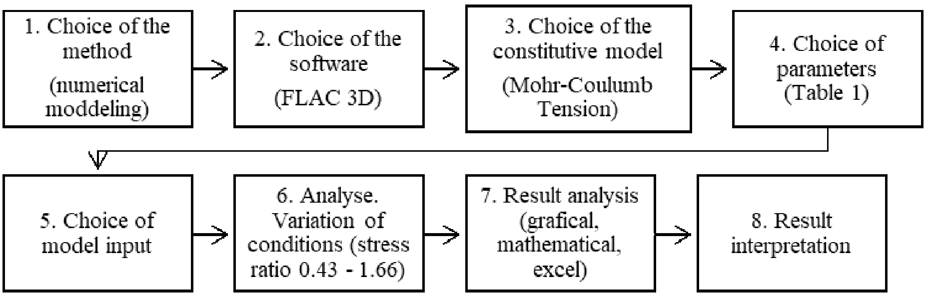


Fig. 1. Methodology chain

1, 2. Method and Software: The numerical solution is performed by the software (FLAC 3D). The choice of numerical modelling is made to avoid external influences by investigating the controlled environment. This is necessary since in reality uncertainty about the input parameters can be higher than permissible output error. To model a geomechanical process, decisions of model parameters as a constitutive model, grid size, etc., must be made. Relative information is provided below (in the parts 3–4).

Limits according to the numerical simulation: As with any research based on numerical experiments, this paper is limited by the approximation included in the numerical methods (Helmut 2008). Despite considering the numerical solution as the most reliable method, the result is sensitive to the decision of researcher as well as to simplification and interpretation hidden behind the material model (Gargani et al. 2006). The limits concern numerical solutions and is not within the scope of this research, although forces the further validation of the theory in the field.

3. Choice of constitutive model: The basis of any constitutive model is the stress–strain dependency. The form of the function in rock mechanics under normal conditions can be divided into elastic and plastic parts.

An elastic model is a model characterized by reversible deformations upon unloading; the stress–strain laws are linear and path-independent (e.x. aforementioned Berry's analytical solution), since it is limited to reversible deformations, it is not the best model type in the case of real rock masses (Lekhnitskii 1963).

The plastic model is a model that involves the nonlinearity of stress–strain relations and is path-dependent. The plastic model can vary. The classical for rock mechanics Mohr–Coulomb model does not cover the cracking and relative stress distribution process (Lekhnitskii 1963). The constitutive model "Mohr–Coulomb Tension" (implemented by FLAC 3D) has been adopted, since it covers tensile failure and closing the tensile cracks influence. The input parameters are located in Table 1; they are representative parameters for a hypothetical sedimentary rock type.

Table 1. Rock mass characteristic

Characteristic	Magnitude		Characteristic	Magnitude	
Density	1600	kg/m^3	Tension	2000	N/m^2
Bulk	4 e7	Pa	Stiffness-normal	4 e8	N/m
Shear	3 e7	Pa	Stiffness-shear	4 e8	N/m
Cohension	1 e20	Pa	Friction	30	grad

4.1. Geometry parameters and boundary conditions: For the hypothetical experiment relative parameters depth to width to height in relation (h:w:m) ~ (40:10:1) with stiff boundary conditions have been used. The influence of the border conditions is less than 0.1% (to estimate that, the distance to borders was increased and compared the difference of results).

4.2. Tectonic condition: The tectonic conditions are applied in the form of a stress ratio $\frac{\sigma_\upsilon}{\sigma_h}$, σ_υ is the vertical stress, σ_h is the horizontal stress. In order to avoid the overvaluation of stress contribution to subsidence troughs due to extreme and rare values, the range stress ration has been taken between 0.43~1.66, the cases are equally distributed around Case 1.

3.2. RESULTS

The results of modelling subsidence for the hypothetical case of sedimentary rock under different tectonic conditions are presented below. Figure 3 present subsidence curves, Figs. 4 and 5 show the relative value of the changes to the tectonic conditions. In figures and in tables the curve named as "Case" with a number related to the stress ratio $\frac{\sigma_\upsilon}{\sigma_h} \in$ (Case 0.43, 0.71, 1, 1.33, 1.66). The curves called "Case–deviation" present the relative value to Case 1 in %.

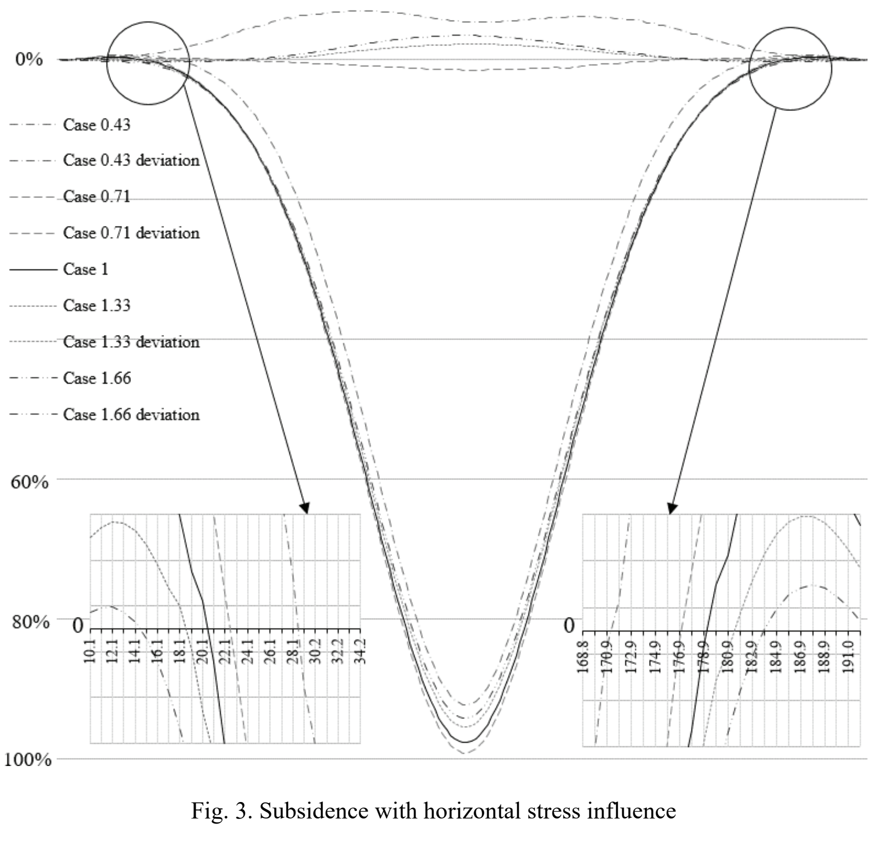
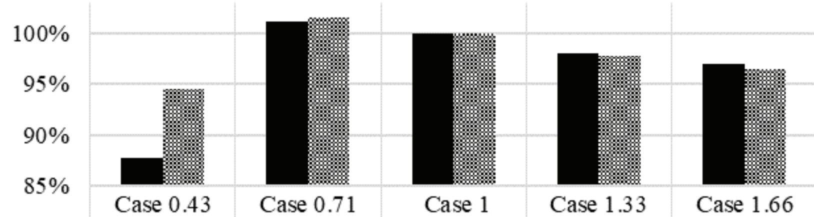


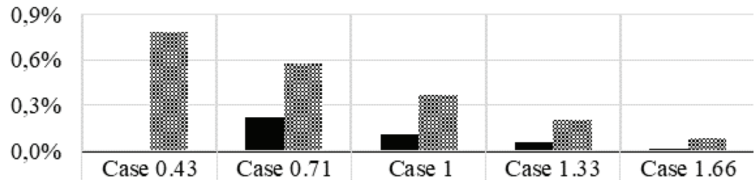
Fig. 3. Subsidence with horizontal stress influence

Figure 3 demonstrates the relative difference in subsidence profiles. The deepest curve belongs to the case with the stress ratio 0.71 (The explanation of the position Case 0.43 is given in the part of Interpretation), with increasing stress ratio the angle of influence increases as well as a maximum of subsidence.



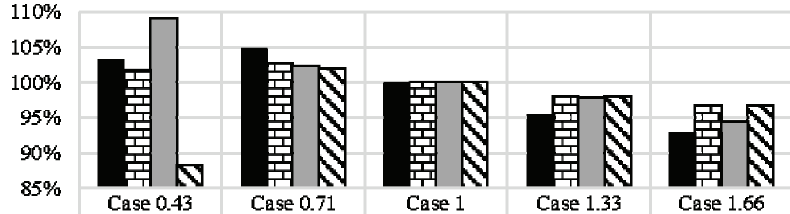
	Case 0.43	Case 0.71	Case 1	Case 1.33	Case 1.66
Volume of subsidence, %	87,73%	101,17%	100,00%	98,05%	97,00%
Max of subsidence,%	94,48%	101,51%	100,00%	97,77%	96,47%

Fig. 4. Changes of subsidence maximum and value of subsidence relative to Case 1



	Case 0.43	Case 0.71	Case 1	Case 1.33	Case 1.66
Uplifting volume/Subsidence volume, %	0,01%	0,22%	0,11%	0,05%	0,01%
Uplifting max/Subsidence max, %	0,78%	0,57%	0,37%	0,21%	0,08%

Fig. 5. Changes of uplifting. Value of uplifting relative to the subsidence value, maximum of uplifting relative to the maximum of subsidence



	Case 0.43	Case 0.71	Case 1	Case 1.33	Case 1.66
Deformation, %	103,3%	104,9%	100,0%	95,6%	93,0%
Tilt, %	101,8%	102,6%	100,0%	98,0%	96,6%
Angle%	109,0%	102,3%	100,0%	97,8%	94,5%
Exploitation coefficient - a, %	88,2%	102,0%	100,0%	98,0%	96,7%

Fig. 6. Changes in deformations and tilt. Deformation, tilt and difference of influence angles from both sides relative to Case 1

Figures 4, 5, and 6 demonstrate changes in main subsidence parameters depending on the applied stress ratio relative to the Case 1. There are demonstrated subsidence characteristics and common subsidence characteristics and their trends according to the applied stress ratio, relative deviations are presented in Table 2. The graphical interpretation makes obvious dependency characteristics used in the subsidence prediction on tectonic conditions. The increasing horizontal stress in general decreases the subsidence predictions characteristics.

Table 2. Variability in the parameter of subsidence with a variation of the ratio $\frac{\sigma_\upsilon}{\sigma_h}$ between 0.43 and 1.66

	Volume of sub, %	Max of sub, %	Up/Sub volume, %	Up max/ Sub max, %	Deformation, %	Tilt, %	Angle, %
Min	87.7	94.5	0.0	0.1	93.0	96.6	96.7
Max	101.2	101.5	0.2	0.8	104.9	102.6	108.9
Δ	13.4	7.0	0.2	0.7	11.9	6.0	12.2

The results show that all parameters change due to the ratio $\frac{\sigma_\upsilon}{\sigma_h}$ variability, up to 8% of the subsidence maximum and up to 12% of the influence angle. In the same time in western Germany, Hegemann (2020) reported deviation from subsidence predicted characteristics – up to 12.3% of the subsidence maximum and – up to Δ 13% of the influence angle, what confirms of the magnitude of the numerical experiment. According to the comparable magnitude of deviation from real cases and investigated influence, the residual uncertainty of prediction methods may be decreased by considering the tectonic conditions.

3.3. INTERPRETATIONS

The plotting of the result demonstrates that in general higher horizontal stress results into a smaller radius of the subsidence trough and a larger volume of subsidence. The unexpected difference between Cases 0.43 and 0.71 leads to a discussion of the influence angle “γ” and exploitation coefficient “a”.

The angle of influence controls the trough width without significant changes in the volume. Thus, the wider the subsidence trough, the smaller the maximum subsidence. It was hypothesised that the higher the horizontal stress, the narrower the trough, which is confirmed by the numerical simulation (Figs. 3 and 5). Therefore, the criteria behaves as expected.

1. Logically the narrower the trough, the smaller the field of void transmission, because through a larger field one would expect larger losses from the void

transmission. This explains the exploitation coefficient increase due to horizontal stress growth.

2. The transformation from horizontal stress to strain may induce higher friction against vertical displacement. Contrary to the first factor, one would expect the exploitation coefficient to decrease with the horizontal stress growth.

Therefore, it is supposed that the first factor has a prime influence in the range of stress ratio 0.71–1.66; further decreasing to 0.43, an increase impact of factor 2. Thus if λ is an angle between normal stress and horizont (Fig. 4): if λ > 23–35 grad – increasing of horizontal stress (decreasing $\frac{\sigma_\upsilon}{\sigma_h}$) increases the exploitation coefficient; if λ < 35–23 grad – increasing of horizontal stress (decreasing $\frac{\sigma_\upsilon}{\sigma_h}$) decreases the exploitation coefficient (Fig. 2).

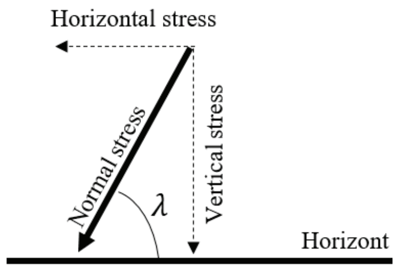


Fig. 2. Visualisation of λ angle, between normal stress and horizon

In general, the high level of confrontation between normal stress and an inner friction angle increases the exploitation coefficient alongside the horizontal stress growth, after which, at a certain critical point it decreases.

4. DISCUSSION AND CONCLUSIONS

The article is devoted to the pre-investigation of a horizontal stress influence on the mine subsidence results. Due to limit to one observed example and the simplification of the geological structure as homogeneous, results cannot be generalized. However, it is sufficient to understand the influence character of the horizontal to vertical stress ratio. The article emphasises the following aspects:

1. Most of the current prediction methods are limited to symmetry and exclude uplift.
2. The main stress fields must be considered by estimation of subsidence prediction parameters.

3. The tectonic conditions are likely to cause deviations from expected subsidence profiles. According to the numerical simulation, changes in stress ratio lead to a maximum deviation of subsidence profile parameters in observed cases. According to the result the deviation of exploitation coefficient (maximum of subsidence) up to 7% and a deviation of the influence angle (influence radius) up to 6% respond to different tectonic conditions
4. The tectonic force could be a reason for the insignificant uplift (0.1% of maximum subsidence) effect, although it can be an influence of the constitutive model and requires further investigation.

The experience shows that changes in the stress conditions bring about changes in the subsidence profile. The dependency requires further investigation, in particular, the smaller subsidence maximum of Case 0.43 in the case of a larger influence angle (discussed in the previous chapter). The stress ratio in the research presents equilibrium conditions and accordingly cannot be a reason for asymmetry. The case of asymmetry due to the non-uniform stress field is left for further investigation.

The results encourage the inclusion of tectonic conditions in subsidence prediction methods. Further research would serve to improve the quality of prediction by the correction of the subsidence parameters caused by tectonic conditions. This includes the investigation on the base of different rock types in wider tectonic conditions to generalize the contribution of the stress conditions to the subsidence profile. The implementation and validation of the theory in the field are left for further investigation as well.

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