

VINTAGE ANALYSIS AS A BASIC TOOL FOR MONITORING CREDIT RISK

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Abstract. Among many tools used by bankers in the process of credit risk management, vintage analysis is the most often applied. Its simplicity and clarity of interpretation of the results means that banking professionals call it “basic analysis”. In this article, the concept of vintage analysis is presented, along with the right way to get interpretations of results. The author demonstrates the usefulness of vintage analysis in the context of the back-testing procedure recommended in New Basel Capital Accord. In addition, there is also a discussion of an aspect of the limit’s structure as a part of the credit risk management process.

Keywords: credit risk, retail loans, probability of default.

JEL Classification: C13, C16, D81, G21, G32, G33.

1. Introduction

Bank employees took the term “vintage” directly from the world of wine. For a long time wine gourmets have been evaluating their noble beverages based on both the species of grapes and the year in which the grapes were harvested. Exactly the year in which grapes ripened has gigantic meaning for the depth of wine’s taste. It is known that poor insolation during a vegetative season in a given year can be a cause of the low sugar content, which in consequence has a gigantic influence on the final taste. Thus, for many years experts have been creating vintage tables, from which one can read a note determining the quality of a given wine from a particular year. Finally, based on a drawn up table, we will know whether a given wine should be stored longer in order to get the optimum taste, or if it should be drunk, or what is worse – if it should have been drunk much earlier.

Wine lovers, who are quite common among bankers, have noticed some analogy between the variable quality of the wine from a given year and

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variable in time quality of credit portfolio built by the bank in a given year. It turns out that loan production performed in a given time can be successfully described with the use of vintage tables.

Bank vintage analysis is now a popular tool for managing credit risk. Due to its considerable functionality, it allows for the ongoing monitoring of risk level, as well as historical levels, providing a complex image of the phenomenon's course in time. Therefore, it often constitutes a starting point for further analysis based on the analysis of time series, providing a broad spectrum of opportunities of analytical risk assessment.

In the literature on the subject, the vintage analysis has been frequently presented as a universal source of knowledge on many areas related with the risk management process. Vintage analysis is recommended by the Experian company, which is one of the world biggest consulting firms specialising in credit risk management processes.

Burns, Stanley (2001) in their paper emphasise the advantages of vintage analysis regarding risk management of retail loans. The usefulness of vintage analysis is presented also by Breeden (2004), who proposed its use in the stress testing analysis. Anderson (2007) dealt with the issue of applying vintage analysis in the process of assessing the accuracy of forecasts. The author concentrated in his paper on analysing the discrepancies between the forecasts of a number of default loans and their actual realisation. In his paper, Zhang (2009) drew attention to the probable character of a vintage curve, whose changeability is significantly influenced by macroeconomic factors. In addition, he proposed a formal model of risk development using the Gaussian process.

It appears that vintage analysis may be applied in the process of estimating capital requirements. In their paper, Ash et al. (2007) considered various approaches to the issue of estimating capital requirements on account of credit risk regarding credit card portfolio. At the same time, they drew attention to the problem of the appropriate segmentation of credit portfolio. In the mentioned paper, the authors were researching two kinds of approach to segmentation. In the first of them, one was analysing fixed segments determined based on vintage analysis, where division was made according to the time of portfolio origination. In the second case, segmentation was conducted based on scoring points, which were determined every certain period. As a result of the obtained research studies, they noticed a significant correlation between the length of portfolio life and the probability of default, which speaks for the application of vintage analysis. However, this correlation is significantly weakening when credits are grouped due to scoring assessment. In addition, Siddiqi (2006) referred to

the possibility of using vintage analysis in many fields. In his paper, he noticed that vintage analysis classically used in the process of detecting high-risk segments and tracking development of this risk may be widened by the application of vintage analysis in the process of constructing scoring cards. The author indicated the possibility of using the curve of risk development estimated based on vintage analysis in order to determine the average length of period in the course of which credit risk increases intensively. This value serves the selection of the training sample used for the estimation of scoring model. In this way, one avoids the danger consisting of taking into account too young loans in the course of estimating parameters of a model whose quality could be unfavourably changed.

Ashcraft, Goldsmith-Pinkham, Vickery (2010) presented the possibility of applying vintage analysis within the framework of early warning system in the paper. The authors analysed mortgage loans granted in the USA between 2001 and 2007, which became the reason for the crisis caused actually by subprime loans. In their conclusions, they drew attention to the fact that when conducting vintage analysis on data from before the crisis, it could be noticed that ratings of subprime portfolios do not reflect the actual risk level.

This paper aims at the presentation of the essence of vintage analysis and of the example illustrating its basic function applied in banking. The author refers also to the definition of default accepted by the Basel Committee on Banking Supervision. Exemplary data are to draw the reader's attention to the wide possibilities of early detection of the changes of the risk level in a credit portfolio. The author wants also to indicate the place of vintage analysis in complex risk management system including such elements as back-testing procedure and system of risk level limits in a bank.

This paper consists of the introduction, after which the concept of vintage analysis is presented, conducted usually in two variants. Further, the author refers to the issue of the so-called assembled vintage analysis enabling the creation of risk development forecasts. Then, there is an example illustrating the results gained with the use of vintage analysis, after which the author refers to the issue of creating risk limits with the use of vintage analysis and conducting back-testing analysis. This paper concludes with the results summarising the advantages of vintage analysis.

2. The concept of vintage analysis

Vintage analysis is usually conducted in the valuable variant, referring to the outstanding of a given loan, and in the quantitative variant, analyzing

the number of sold loans. Valuable analysis consists in setting the matrix of indicators constituting the quotient of the capital balance of a given loan and the original value of the granted loan capital. The quantitative variant refers, though, to the number of loans observed in individual risk groups.

The essence of this analysis is the presentation of a given portfolio risk level where the portfolio is grouped with regard to two characteristics. The first characteristic, the so-called invariant, is an attribute of either a borrower or a loan, whose value is known at the moment of granting a loan, and which remains constant in the bank system. Similarly to wine methodology, a year of granting a loan, but also its value or borrower's sex can be an example of such a feature. Usually these characteristics constitute rows of the grouping table. An indicator of risk is always the second characteristic of vintage analysis. In the case of consumer finance sector, the risk indicator is usually determined by the number of days of delay in the repayment of debts. This last feature constitutes the basis of grouping loans, thereby setting columns of the table. Risk indicators are determined when there is two-dimensional space.

The primary aim of vintage analysis is the presentation of the credit risk development of a given portfolio in order to enable tracking its trend of development and its further anticipation. The analysis thus provides not only current information, but also historical, which allows constructing forecasts based on models of time series. An important feature of applied statistical models is the possibility of a comparative analysis of the risk level with respect to different values of particular characteristics of a loan or borrower. In its complexity, vintage analysis allows obtaining valuable information, with particular consideration for:

- comparison of risk level in particular months/quarters/years,
- analysis of the influence of particular characteristic's value on the credit risk,
- analysis of the influence of the internal risk policy changes on the portfolio risk,
- forecasting the risk level in the future,
- current monitoring of the portfolio risk level.

Therefore, vintage analysis is one of the basic analyses used for measuring a risk in the process of managing it. It constitutes a starting base for formulating conclusions and forming the general policy of credit risk management. Correctly interpreted results allow detecting not only dangerous increases in the risk level, but, what is more important, they allow locating sources of the risk increases.

As was mentioned above, vintage analysis focuses on the estimation of the series of risk indicators constituting in the valuable variant quotients of overdue loan capital from the tested group with determined delay in repayment and at the set value of invariant feature against the initial value of granted loans at the set invariant feature. Therefore, the denominator is always the original value of granted loans of the entire group for a given value of invariant characteristic regardless of the value of second attribute characterizing the risk level. In the quantitative variant, values of outstanding capital are replaced with numbers of granted loans.

Further in this paper, principal of loans (outstanding) will be denoted by the letter K . Every time when an indexation $K_{i,j}$ appears, it will mean the value of outstanding capital of all loans granted at given value of invariant characteristic I and at given value of the characteristic describing risk – j . For example, $K_{2005-01,0-30}$ means total outstanding for all loans granted in January 2005, whose delay in repayment does not exceed 30 days.

Figure 1 shows an example of a table containing values of the outstanding capital $K_{i,j}$. In this classical approach, the invariant feature is the month and year of granting a loan. Indicator of risk is then DPD (Day Past Due).

		Initial value of granted loans	Day Past Due							repaid loans	loss on the sale of loans
			(0 - 30^>	(30 - 60^>	(60 - 90^>	(90 - 120^>	(120 - 150^>	(150 - 180^>	> 180 days		
Date of Granting Loans	2005-01	$K_{2005-01}$									
	2005-02	$K_{2005-02}$									
	2005-03	$K_{2005-03}$									
	.										
	.		Outstanding								
	.										
	2007-01	$K_{2007-01}$									
	2007-02	$K_{2007-02}$									
	All										

Fig. 1. Example of a vintage table – valuable (nominal) approach

Source: author’s own study.

Based on $K_{i,j}$ values, there are set $V_{i,j}$ indicators determining the share of remaining principal $K_{i,j}$ in the initial value of granted loans – at a given value of invariant feature i and at all possible values of j feature. Therefore, we have:

$$V_{i,j} = \frac{K_{i,j}}{K_{i,\bullet}},$$

where:

$K_{i,\bullet}$ – is a total amount of lent capital in the moment of granting for a group of loans with given value of i feature;

$V_{i,j}$ – risk indicator in the valuable variant for given value of i invariant feature and at given value of j risk feature.

In the quantitative variant, analysis is conducted in the analogous way with use of the following notation:

$L_{i,j}$ – number of loans granted at set values of i invariant feature and j feature describing risk.

$$N_{i,j} = \frac{L_{i,j}}{L_{i,\bullet}},$$

where:

$N_{i,j}$ – risk indicator in the variant of the quantitative analysis for a given invariant feature i and at given value of j risk feature;

$L_{i,\bullet}$ – number of loans granted at a given value of i invariant feature.

3. Indicator of risk in vintage analysis

According to the above-mentioned considerations, the risk level indicator in vintage analysis is DPD. An index assigned in risk indicators is letter “ j ”. Basic risk analysis is based on the following grouping of the number of days of delays (DPD):

1. from 0 up to 30 days;
2. from 31 up to 60 days;
3. from 61 up to 90 days;
4. from 91 up to 120 days;
5. from 121 up to 150 days;

6. from 151 up to 180 days;
7. above 180 days.

For a full reflection of the risk level, we can also use the next two values of the risk feature:

8. Value of the principal being repaid, or in the quantitative variant, number of loans with fully repaid principal.

9. Value of principal lost on closed loans (or sold as a part of purchase and sale of receivables), or in the quantitative variant, number of loans that have not been repaid completely.

Category no. 8 in the valuable variant is complementary to the possible situations of loans with regard to risk. After all, at the end of the portfolio life, we receive two possible classes of risk. The first one is obviously a group of repaid loans, and the second group is so-called “lost loans”. The analogous situation occurs in the quantitative variant of analysis.

Category no. 9 in the valuable variant illustrates this part of principal, whose recovery is impossible from the economic point of view, or there is already no legal title on the bank side to start debt collection actions. This situation concerns these debts which due to long-term arrears in repayment were sold to special funds occupied with long-term debt collection actions. According to the vintage analysis, profits from the sale of these debts increase the value of repaid principal (category no. 8). In recent years, the value of debt recovered in this way has constituted a greater and greater part of the recovery rate. More and more often, banks decide to refrain from the strategy consisting in long-term recovery of individually relatively low amounts. In this area, the sale of debts turns out to be more effective. Cash obtained in this way strengthens bank’s liquidity and allows conducting further credit activity. The value of debts depends on the period of arrears and intensity of previous debt collection actions called “portfoliodraining”. In recent years, it has been possible to observe many transactions of selling overdue debts, whose value for the above-mentioned reasons fluctuated from 10% to 20% of the value of the outstanding amount.

In the quantitative variant of vintage analysis, category no. 9 includes a number of loans which, in accordance with the valuable approach to this category, are not subject to the further process of recovery.

The inclusion of categories 8 and 9 in vintage analysis allows for a broader look at the problem of credit arrears. Analyzing a selected row of a table, referring to, e.g. loans granted in a given year, we obtain a full spectrum of situations, in which there is capital lent to borrowers. Taking into account sold debts in vintage analysis, we are reaching not only for

so-called “active loans”, but also for those left in the banking book. This allows for the recognition of the current situation for 100% granted loans analyzed with regard to credit risk.

4. Assembled vintage analysis

In addition to static vintage analysis conducted for the specific day with taking into consideration the loan situation and their balances on the day of analysis, bankers build so-called “risk development profiles” consisting of many isolated vintage analyses. They allow creating forecasts of the credit risk level in the future. These forecasts are built with use of statistical analysis of time series. Therefore, it is so important to design a data warehouse in the banking system so that it is possible either to achieve historical results of vintage analysis or reconstruct these analyses for any date in the past.

A significant difference between the terms of analysis date and database date should be emphasized here. Every conducted analysis includes both dates. The analysis date is the day on which the given analysis was conducted, and the database date is the day according to which the values of the loan balances were considered. For example, we can imagine analysis conducted today, but referring to the state of system from last month (database date). Certainly, the analysis date cannot be older than the database date.

Traditionally, bankers forecast firstly the percentage of lost loans in the total value of granted loans. As lost loans one usually considered those receivables whose delay in repayment exceeds 90 days, and whose value of arrears exceeds the threshold amount. The threshold amount is set within the framework of internal bank policy and usually oscillates within the limit of 50 PLN for retail loans. Its task is to exclude from risk analysis all these loans whose arrears are caused by underpayment resulting from FX ratechanges (in the case of foreign currency loans), or from the change of installment amount because of a change of interest rates. In addition, banking practice consisting of accounting in the first place costs of reminder letters and reprimands which causes a small underpayment of the principal amongst the borrowers using the system of direct debit. The threshold amount has also a pragmatic function due to the way of DPD calculation used in the banking system. Usually bankers calculate a threshold number of 90 days defining the case of insolvency from the day of its appearance. For example, let us consider a situation when the borrower was charged for a reminder letter with the amount of 10 PLN as a result of a three-day delay

in repayment of installment worth 500 PLN. If the borrower pays the full installment (500 PLN) and forgets about paying the cost of the reminder letter, then the majority of banking systems will cover firstly the reminder letter cost. In the next step, banking systems allocate the remaining amount to cover the penalty interest specified in the credit agreement, then the loan interest, and at the end, the principal. At this moment, the borrower is still having arrears of principal worth 10 PLN (without taking into account the penalty interest calculated for the above-mentioned three-day delay). Thus, in spite of further regular payments of full principal-interest installments, this arrear is still active. What is the most important is the fact that the date of arrears remains unchanged from the moment when the borrower was late with the installment for the first time (the above-mentioned three days). Therefore, taking into account the accepted rules of calculating days in arrears, bankers should analyze this effect in order to improve the estimation of the credit risk.

In order to obtain the risk profile, the bankers must prepare the time series for credits granted in closed periods, usually monthly, but also quarterly or annually.

Figure 2 shows the concept of building a data structure for loans grouped in monthly periods with regard to a granting date.

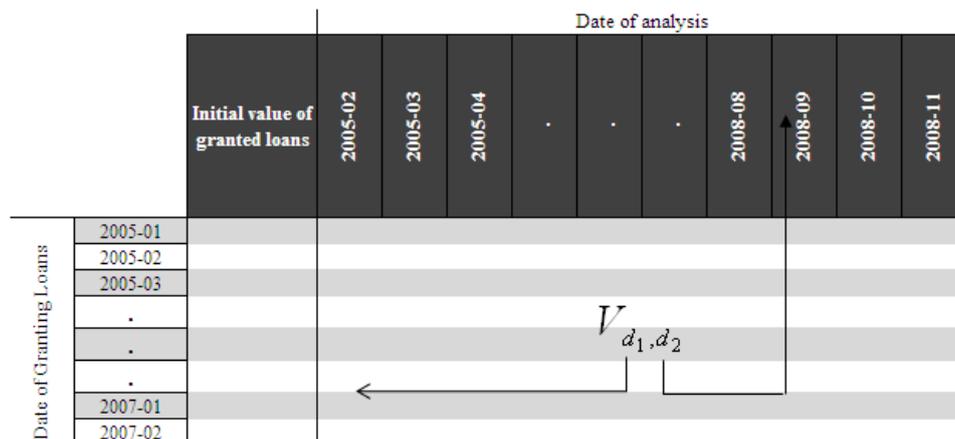


Fig. 2 Model of a table of the risk profile analysis – valuable approach (percentage)

Source: author’s own study.

Particular rows represent risk indicators $V_{i,j}$ for loans granted in i month, where j index represents the value of the indicator according to the

date of bank system. Usually dates of banking systems are set at the end of months. Figure 2 presents an example of a table containing the risk profile for credits granted from January 2005 to February 2007. The time series of the development of risk profile, in this example, covers the period from February 2005 to November 2008.

Based on tabular data prepared according to the above-mentioned way, presentation of a trend of risk level's development is possible for particular groups of loans granted in consecutive periods. This data constitute the basis for further analyses that allow anticipating the risk level in the future.

The following graph presents exemplary profiles of risk, understood as a percentage of lost principal (DPD > 90 days) in the value of granted loans.

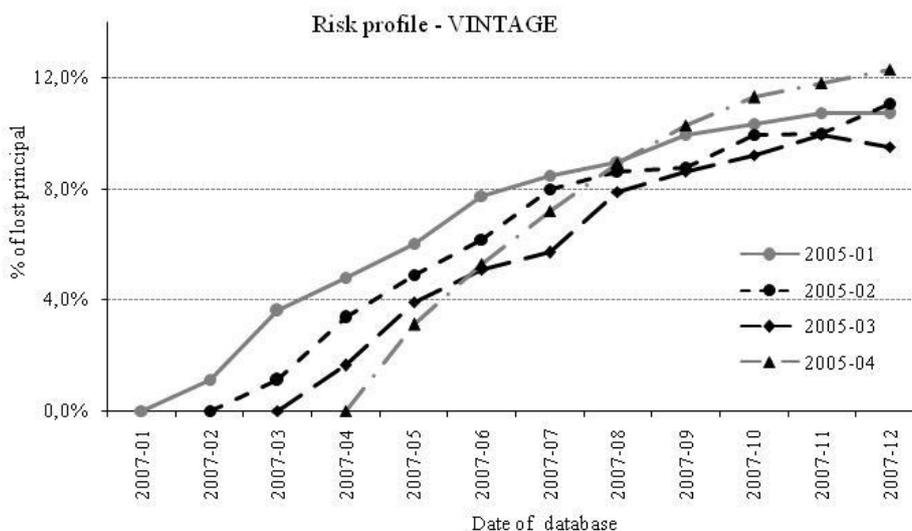


Fig. 3 Presentation of risk profiles for individual portfolios

Source: author's own study.

Another extremely useful variant of the above-described analysis is the presentation of tabular data in a layout that allows comparing different risk profiles easily. We make it by replacing column headlines containing database dates, presenting values of risk indicator on a given day by following numbers indicating the length of portfolio life (in months). In fact, data given in the table (see Figure 2) require transformation consisting in moving particular rows left by one period with regard to the higher row. This trans-

formation starts from the second row. After such an operation, required setting months of portfolio life is required. This enables the comparative analysis of risk level for particular portfolios. The discussed procedure concerns certainly credit portfolios grouped with regard to the month of granting. In the case of analysis of quarterly or annually grouped portfolios, we should take into account the appropriate shift interval.

Figure 4 presents an exemplary table illustrating the development of risk profile of portfolios in the comparative approach:

		Initial value of granted loans	number of months from the beginning of life												
			1	2	3	.	.	.	13	14	15	16			
Date of Granting Loans	2005-01														
	2005-02														
	2005-03														
	.														
	.														
	.														
	2007-01														
	2007-02														

Fig. 4. Model of a table of the risk profile analysis – comparative variant

Source: author’s own study.

According to the above-described concept allowing for easier comparison of risk profile development, data layout needs to be transformed. Because of this transformation, particular portfolios represented by lines are analyzed according to length of life measured in months from the granting date. Such a presentation allows tracking the trend of development of risk, and, what is more essential, it allows for the direct comparison of risk indicators within particular columns. According to this approach, the created graphs allow tracking the impact of changes of bank credit policy on the level of risk as well as the impact of macroeconomic factors on the risk. In addition, the following periods of portfolio life are subject to comparison with each other. Figure 5 provides a graphical presentation of risk’s trend of development for several months in the comparative variant.

In Figure 5 we can see the approximate profile of risk development for loans granted in months: 2005-01, 2005-02 and 2005-03. The portfolio of loans granted in the month 2005-04 has a different shape of risk profile.

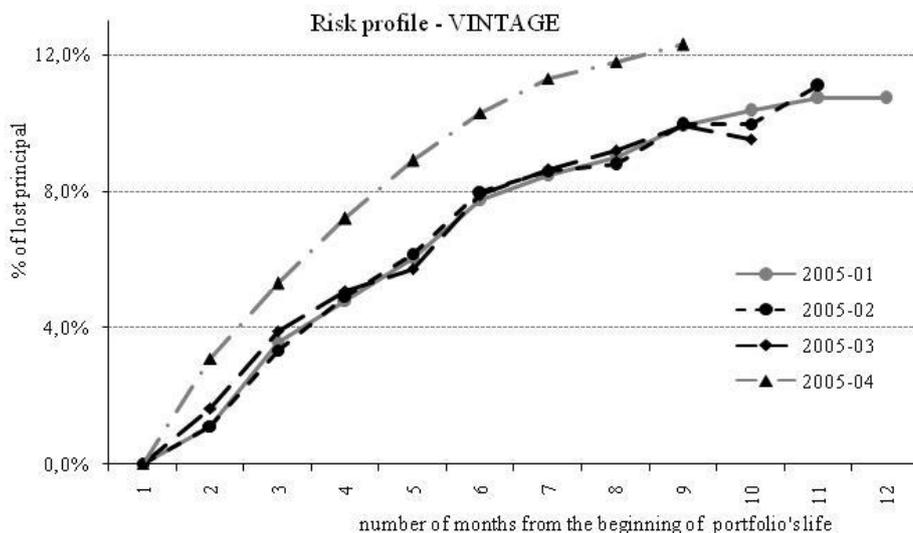


Fig. 5. Presentation of risk profiles for particular portfolios – comparative variant

Source: author's study.

The observed discrepancy between particular rows representing risk levels for credit portfolio is a rich source of information about the internal risk management policy and direction of the credit risk development. The significant increase of risk may arise from the liberalization of the bank's approach to risk management policy, it can also result from changes of macroeconomic factors significantly determining the quality of loan portfolio. Regardless of the reason of the observed discrepancies, especially in comparative variant of vintage analysis, it may serve as an early warning system in a situation where each month represented by the row of another color (see Figure 5) is above the profile of the previous month.

5. Vintage analysis vs. back-testing

Within the framework of validation procedure of risk estimation models applied in banking, verification of obtained forecasts is conducted by comparing them with actual losses incurred by the bank. The bankers define the estimation of the accuracy of forecasts as the back-testing analysis. The Basel Committee on Banking Supervision has given it special significance. The Committee has established its place in the overall process of credit risk

management. One of the basic parameters of risk assessment model provided by the New Basel Capital Accord, which is standard for capital requirements calculation, is the probability of default – PD. Recommended by the Basel Committee definition of default, with regard to retail loans, it is based on the indicator of arrears in the repayment of loans at the level of 90 days (90 DPD Day Past Due). Also the recommended forecast horizon comes to one year. That is why it makes the vintage analysis a useful tool to conduct back-testing procedure of the PD model applied in the bank.

For this purpose, vintage analysis is used in the quantitative variant. It includes the analysis of the number of loans for which the bank determined the occurrence of default. Based on these data, it is possible to confront the obtained values of forecasts with actual realizations of the risk. Thus, vintage analysis perfectly complements the comprehensive system of credit risk management and constitutes its important element.

6. Limits of the credit risk

In traditional retail banking, credit risk is the most important element of banking risk. Therefore, it is subject to special supervision as a part of internal control system in every bank. The series of disgraceful events that resulted in the bankruptcy of many banks in the past two decades revealed gaps in the risk supervisory systems. Many of the spectacular bank failures in recent years could have been avoided if only the supervision of persons directly generating open positions, constituting the source of risk, had been conducted properly. The process of limiting risk factors is an effective way of exercising supervision. Typically, bankers determine the critical values of risk factors which are safe for the bank. These limits refer mostly to the size of the open position maintained by dealers. A properly configured computer system makes it virtually impossible to conclude transactions resulting in exceeding the specified limit. In the case of credit risk, the issue is much more complicated than it is for example in the FX risk or interest rate risk. This is due to the nature of credit risk, which appears relatively much later after the opening of the bank's position (loan granting). Nevertheless, in the credit risk management system so-called "instance decision-making process" should be implemented. It consists of indirect informing appropriate bank units in the case of exceeding previously set risk limits. In banks with a high culture of risk management, the head of a department responsible for the area of credit risk management is required to take immediate actions whenever any of the risk indicators reaches the alarm level. These alarm

levels are determined by a resolution of the bank's management board. In the case of exceeding the set limit, the first action is the obligation of passing reliable information about the level of risk and, obviously, its reasons both to the bank's management board and to the asset and liabilities management committee. At the same time, on application of the head of risk area, the ALCO (Asset-Liability Committee) committee recommends the necessary remedial actions to the board of the bank.

A key element in the system of limits is naturally the method of selecting risk indicators that will effectively control the level of risk, to which each bank is exposed. The indicators presented in vintage analysis, i.e. N and V , are particularly useful. Due to a percentage formulation, they are a convenient tool to compare reliably the risk level with the previously approved limit. The value of limits adopted by the bank's board should be obtained from the risk appetite. The estimation of risk appetite is another issue that requires thorough consideration.

7. Summary

The world of banking, which has been shaken by the latest global financial crisis, is still struggling with the problem of standardization of risk measuring methods. Barely implemented standards in the area of capital requirements recommended under the New Basel Capital Accord cause controversy reflected in the currently consulted accord – Basel III. More and more sophisticated methods of risk management, which are currently being developed in modern banks, are unquestionably the right direction toward improving the safety of the banking system. However, no one can forget about good, relatively simple approaches to risk management, which undoubtedly include vintage analysis.

Moreover, it must be remembered that each bank is a structure consisting of people with different perceptions of the environment. This concerns particularly such a complex area as credit risk. The organizational structures of financial groups have been evolving into a collective key decision-making approach. In these collective teams, there are both people of great experience in the field of quantitative area and those who have less knowledge in this area. Thus, it is so important that during the decision-making process all team members have reliable and clear information. The simplicity of vintage analysis meets these conditions, constituting the perfect supplement to more complex methods.

The article presents the concept of vintage analysis. The purpose of the article was also to indicate the areas in which vintage analysis can be used. It turns out that it can be implemented both in the analysis of stress test and in the back-testing analysis. It can serve as a tool to indicate the current level of credit risk and it can be the source of forecasts. It can also be used as an early warning system in the bank. The article also highlighted the fact that it is possible to create a system of risk limits based on the results of vintage analysis.

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