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**HOW TO TEACH QUANTITATIVE SUBJECTS
AT UNIVERSITIES OF ECONOMICS IN
A COMPREHENSIBLE AND PLEASANT WAY?**

Helena Gaspars-Wieloch

Abstract. Nowadays in Poland: the level of pupils' and students' mathematical skills is constantly decreasing, students refer very seldom to manuals, because they are not able to understand their content, lecturers working at universities are not obliged to have a suitable pedagogical experience, the number of hours designed for mathematical subjects at universities of economics is smaller than it was in the previous decades, university authorities just recommend reducing requirements towards students in order not to lose too many of them at the end of the academic year.

The rules mentioned in this article are devoted to help Polish lecturers working at universities of economics to teach economic quantitative subjects (e.g. Operations Research, Statistics, Econometrics, Forecasting and Simulation, Project Management) in a comprehensible and pleasant way. If they manage to fulfill these conditions: the atmosphere in class will be more pleasant, economic quantitative subjects will be better understood by the students, the listeners will be more interested in topics presented by the lecturer, grades will be higher.

Keywords: colleges economical, effective and fun teaching, teaching quantitative subjects, teaching tips for teachers.

1. Introduction

I have been working at the Poznań University of Economics since 2003. So far I have had the opportunity to teach Operations Research, Operations Research in Logistics, Econometrics, Project Management and Forecasting and Simulation. That is why, the guidelines discussed in this article will often be illustrated by examples relating to these economic quantitative subjects.

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Some rules mentioned below concern all subjects and are known by everyone for sure. Nevertheless, in my opinion, it is recommended to enumerate them once again because I notice much negligence in this field at Polish universities. Other hints are devoted to help a good quantitative subjects teaching.

All the guidelines may be very helpful for Polish lecturers, especially nowadays when:

- the level of pupils' and students' mathematical skills is constantly decreasing (Trochanowski 2004a; Twardowska 2008),
- students very seldom refer to manuals, because they are not able to understand their content (Goworek 2003),
- students study many faculties simultaneously and they work as well,
- many lecturers are not obliged to have a suitable pedagogical experience (Trochanowski 2004a),
- the number of hours designed for mathematical subjects is smaller than it was in the previous decades,
- the number of topics discussed within the majority of quantitative subjects has fallen down for the last decade,
- university authorities just recommend reducing requirements towards students in order not to lose too many of them at the end of the academic year.

2. Guidelines

1) Your goal is to explain something and not to boast about your abilities

Remember that the lecture hall is not a bandstand! You stay there to help students, to arouse their interest in a given topic, to explain difficult problems to them and to encourage them to study (Smoluk 2004). If you have different aspirations, you should not be a teacher. Unfortunately, in Poland university employees must be scientists and teachers simultaneously, even if they are not fit for one of these occupations. Therefore, in Polish universities one can meet many people having no eagerness to teach or do research.

In order to put the first guideline into practice:

A) Take care of the diction. It is obvious that teachers ought to speak extremely clearly, especially when they introduce new expressions.

Meanwhile, the reality is totally different. For the moment, some lecturers speak under their breath, others do not pronounce each syllable (often the last one). Such attitude among teachers as well as TV announcers, politicians or priests is unacceptable.

B) Do not use too difficult words. Some of us state that using difficult words and complex mathematical formulas increases the prestige of a given course and the respect for the lecturer (Smoluk 2004). Perhaps students will tolerate several such complicated expressions but if they occur too frequently and they are not explained during the course, the participants will just stop listening or stop attending the lecture.

C) Try to find a common language with the audience. If you notice that listeners do not understand your comment, try to explain a given problem one more time using other examples, other drawings, other words.

2) Use presentations prepared by yourself, but do not put the whole content of your course on slides

The teacher who uses presentations prepared e.g. in Microsoft Powerpoint rationalizes a lot the explanation of a given topic. Thanks to the slides:

- our lecture may be more structured,
- we are sure that the listeners will know how to spell difficult words,
- we do not waste our time dictating the subject, writing complicated formulas on the board or dictating the content of an exercise which is going to be solved in a minute,
- we can quickly show a chart, a picture, a film illustrating the problem involved.

Nevertheless, we should beware of putting the whole content of our course on slides! There are two reasons. Firstly, students will come to the conclusion that there is no need to attend such a course. Secondly, even if they decide to participate, they will not understand as much as they would understand if they had to solve a problem step by step with the teacher.

Project network analysis

Keywords:

1. Project 2. Network 3. Event 4. Activity 5. Dummy activity

1. What are the characteristics of an AOA network (activities on arcs)?

2. Why can not the following graph represent the structure of a project?

Important formulas:

(1) $t_i = \max_k(t_k + t_{ki}) \quad i = 2, 3, \dots, n$ (2) $t_i = \min_k(t_k - t_{ik}) \quad i = n-1, \dots, 2, 1$

(3) $Z_{ij}^c = t_j - t_i - t_{ij}$

Operations Research Helena Gaspars-Wieloch

Fig. 1. Introduction to the Project Network Analysis

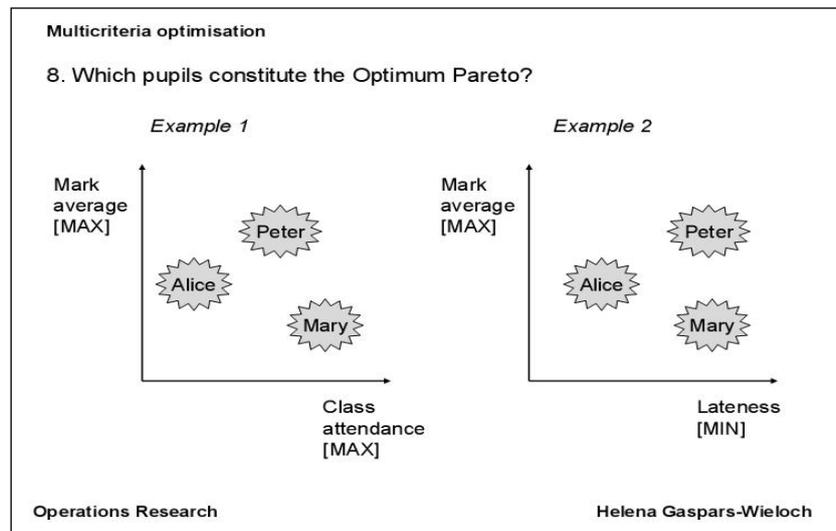


Fig. 2. An exercise concerning the Optimum Pareto

Listeners are more active when the slides include only brief titles that are developed by the speaker during classes. That is why, I suggest that the slides consist of:

- keywords without the whole definitions (see Fig. 1),
- theoretical questions without the answers (see Fig. 1),

- mathematical expressions, charts, pictures without additional explanations (what does this symbol signify?, what does this picture show?, what conclusions can we formulate on the basis of this chart? etc.), see Fig. 1,
- the content of a problem to solve without the solution (see Fig. 2),
- tables and texts with gaps to fill up (see Fig. 3).

3) From time to time you may use something amusing

Our memory is especially visual and situational. Therefore, you may illustrate a problem or explain the application of a given method by introducing funny supplements in your presentation, see Fig. 3.

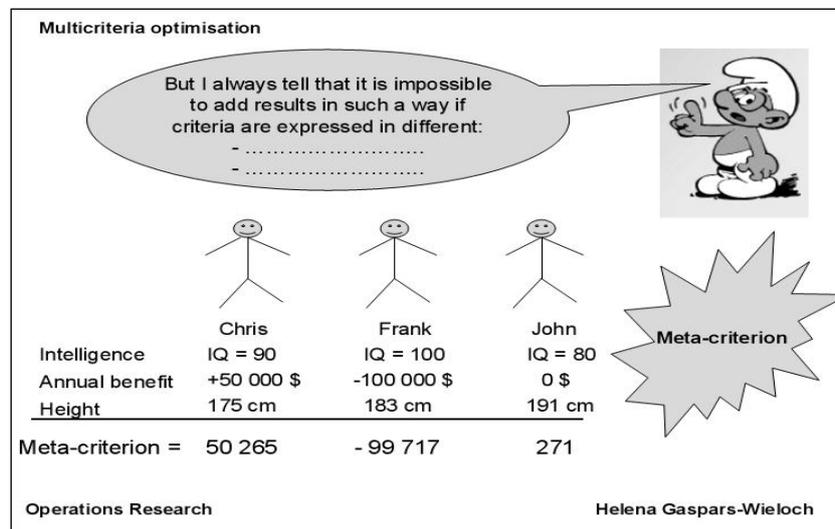


Fig. 3. An exercise related to the meta-criterion

4) Explain first of all what a given method is for, and not how to use it. Do not let students solve problems in a mindless way

When students try to solve a mathematical problem, they often do not actually understand what the goal is, what a given method consists in. Thus, when the exam exercise is formulated a little bit differently than it was during the classes, the majority of them make serious mistakes. In order to avoid such situations we ought to (Nowakowski 2004):

- teach students universal methods,
- explain procedures very precisely,
- discuss diverse examples.

Let us analyze the following cases¹:

A) The optimal solution in the graphic method may be found by means of the extreme points review, the gradient or the isoquant. If we use the isoquant, we cannot tell the listeners that the solution maximizing (minimizing) the objective function is always indicated by the isoquant touching the feasible region in the top-right (bottom-left) corner. This guideline is only useful when both objective function coefficients are non-negative! If at least one parameter is negative, we must compare the objective function values calculated for the potential optimal points and then choose the most profitable one. If we do not present universal hints and do not put them into practice before the exam, students will answer that in the problem formulated below, the extreme point B is the optimum, which is false of course.

$$\begin{aligned}
 (0) & -3x_1 + x_2 \rightarrow \max \\
 (1) & 2x_1 + 2x_2 \leq 12 \\
 (2) & 5x_1 \geq 5 \\
 (3) & x_1 - x_2 \leq 0 \\
 (4) & x_1, x_2 \geq 0
 \end{aligned}$$

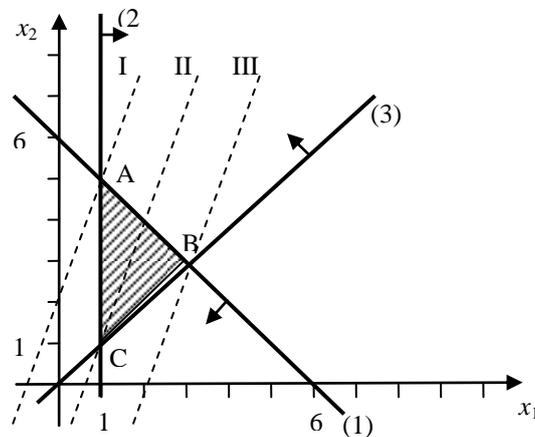


Fig. 4. Searching of the optimal solution with the aid of the isoquant

¹ The two first examples concern Operations Research (Modelling linear decision problems. Geometric method).

B) A universal procedure should also be given when we explain how to draw in the coordinates system the arrow related to an inequality constraint. If we assume that all variables must be put on the left side of the constraint, the hints in Table 1 are correct only when all left-side parameters are non-negative (see constraints (1) and (2))! Otherwise (see constraint (3)) these rules are pernicious.

Table 1. Constraints' marking

Inequality sign of a constraint	Direction of feasible solutions
\leq	
\geq	

Thus, it is much more safe to recommend the following procedure:
 1) Choose a point which does not belong to the line corresponding to the constraint considered (compare Guzik 2009, 2)
 2) If the chosen point satisfies this restriction, all points situated on the side of this point are feasible. Otherwise, these points are inadmissible.

C) The last example concerns the Savage rule (Sikora 2008)². This procedure, depending on its stage, consists in calculating or searching a specific value for all possible strategies (e.g. A, B, C) or within states of nature (e.g. S1, S2), see Table 2.

Table 2. Savage rule

	Revenues				Relative losses			
	A	B	C	Max revenue (↓)	A	B	C	Min max relative loss (↓)
S1	9	5	10	10	1	5	0	
S2	9	12	8	12	3	0	4	
				Max relative loss (→)	3	5	4	
				Optimal strategy (→)	A			

Meanwhile, students prefer thinking about columns and rows rather than about decisions and states of nature. This attitude is quite risky because

² This topic is discussed within the Operations Research courses (Decision-making under uncertainty).

during the exam the table may be given inversely (i.e. decisions might be presented within rows and states of nature within columns)! The task of the lecturer is to explain the idea of this method and not the mechanism. Students must understand that the goal of the Savage rule is to choose this strategy which minimizes the maximal relative loss.

Let us also consider the following linear optimization model³:

$$\begin{aligned}
 (0) \quad & 2x_1 + x_2 + 3x_3 \rightarrow \max \\
 (1) \quad & x_3 \leq 4 \\
 (2) \quad & x_1 + x_2 + x_3 \leq 10 \\
 (3) \quad & x_1 - 2x_2 \leq 0 \\
 (4) \quad & x_1, x_2, x_3 \geq 0
 \end{aligned}$$

Assume that students only know the geometric method as yet. It means that they are able to draw the feasible region and the isoquant in the coordinates system. Notice that this procedure is merely designed for problems containing 2 decisive variables, alternatively 3 variables providing that at least one constraint is an equation. The problem above cannot be solved by this procedure. However, if students understand that the goal of the geometric method consists in finding the best admissible solution, they will try to find such a combination of variable values for which the objective function achieves the highest value and all the constraints are satisfied. In the function (0) the coefficient next to x_3 is the highest and this variable may amount to at most 4 (see constraint (1)). According to the constraint (2), the sum of x_1 and x_2 cannot exceed 6 (assuming that $x_3 = 4$). The restriction (3) means that x_1 may be at most equal to $2x_2$. In the function (0) the variable x_1 has a better coefficient than x_2 , but the constraint (3) allows this variable to equal just 4 and then x_2 equals 2.

This example shows that the student may find the result even if he does not know all necessary procedures related to a given topic. He just ought to think logically and not use a method thoughtlessly without understanding what he is actually doing.

5) If it is necessary, explain vividly the idea of an algorithm

Let us discuss the Critical Path Method (CPM), (Trocki 2003). Students have difficulty in understanding why the shortest completion time of

³ This example concerns Operations Research (Modelling linear decision problems. Geometric method).

a project presented by a network is determined by the longest path (and not by the shortest one)! Figure 5 illustrates the structure of a project⁴. The network is drawn according to the AOA technique (Activities on Arcs). The project contains 5 activities: A, B, C, D and E and their duration equals 5, 4, 2, 6 and 3 time units, respectively. The numbers in brackets represent the activity floats. In this network the longest path consists of A, C and E. Its total duration amounts to 10 units and this is also the shortest project completion time. Participants often ask me why we have to wait so long, since there are shorter paths in the network (A-B = 9 units, D-E = 9 units). In their opinion, there is a possibility to accomplish the project in 9 units. Then I tell them that after 9 units the activity E will be still executed, for it has started only after the 7th unit and it lasts 3 units.

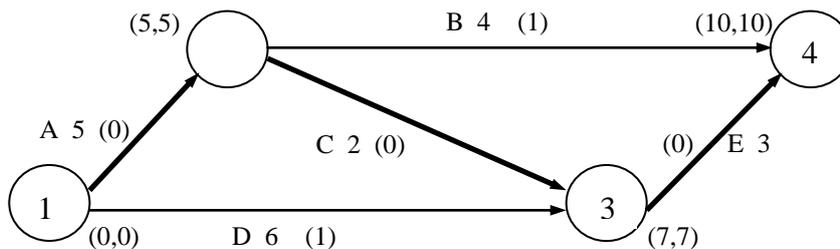


Fig. 5. Calculating the shortest project completion time

I also ask them to imagine that each path represents a runner (path A-B: runner 1, path A-C-E: runner 2, path D-E: runner 3). The path duration signifies the time of a given runner (9, 10, 9 units, respectively). When will the competition officially end? It is obvious that after 10 units.

6) Present particularly these algorithms that may be explained on the basis of real decisive problems. Solve as many real tasks as possible. Avoid irrational methods and useless topics

Students are much more interested in examples related to real problems (Nowakowski 2004; Trochanowski 2004a, 2004b; Żylicz 2009). That is why, you may for instance:

- use newspapers or internet data to prepare a ranking for banks⁵,

⁴ This example concerns Operations Research (Time Project Network Analysis) and Project Management.

⁵ See Multicriteria Optimisation – Operations Research.

- use a real case to explain the Time-Cost Project Network Analysis⁶,
- present the Portfolio Analysis on the basis of current exchange quotations⁷,
- present forecasting methods using current data concerning the unemployment rate, the inflation rate, the foreign exchange rate etc.⁸

On the one hand, quantitative subjects offer a large range of simplified procedures that cannot be applied in real situations. On the other hand, sometimes it is recommended to present at the beginning of the course an algorithm designed for decisive problems with many unreal assumptions, especially when the lecturer wants his students to think up, on the basis of this simplified procedure, a more complex method for real examples. If the presentation of a given algorithm has no didactical or practical value, it is better not to put it forward.

Lecturers should avoid methods which generate irrational solutions. For instance, the presentation of the metacriterion for the continuous multi-criteria optimization does not make sense, especially when we use the formula (1).

$$M = \sum_{i=1}^m w_i f_i(x) - \sum_{j=1}^n w_j f_j(x) \rightarrow \max, \quad (1)$$

where: M is metacriterion, m – number of maximized objective functions, n – number of minimized objective functions, w_i , w_j – importance of the objective function $f_i(x)$ or $f_j(x)$.

This equation is applicable to criteria expressed in the same units and scale.

Let us analyze the following example⁹.

$$(01) \quad x_1 + 7 x_2 \rightarrow \max$$

$$(02) \quad 3 x_1 - 2 x_2 \rightarrow \max$$

$$(1) \quad x_1 + x_2 \leq 10$$

$$(2) \quad x_1 \leq 4$$

$$(3) \quad x_1, x_2 \geq 0$$

⁶ See Project Management and Operations Research.

⁷ See Operations Research, Financial Instruments and Institutions, Investment Portfolio Management.

⁸ See Forecasting and Simulation (Cieślak 2001).

⁹ This example concerns Operations Research (Multicriteria Optimisation).

Table 3. Optimal solutions

Criterion	(01)	(02)
Variable's value	$x_1 = 0; x_2 = 10$	$x_1 = 4; x_2 = 0$
Function value of criterion (01)	70 (max)	4
Function value of criterion (02)	-20	12 (max)

Table 3 presents the optimal solutions for both objective functions. If we assume that, according to the wish of a given decision-maker, the second criterion is three times more important than the first one ($w_1 = 1, w_2 = 3$), then the formula (1) emerges in the form:

$$M = 1 \cdot (x_1 + 7x_2) + 3 \cdot (3x_1 - 2x_2) = 10x_1 + x_2 \rightarrow \max$$

and the final solution is as follows (see Table 4):

Table 4. Final solution

Variable's value	$x_1 = 4; x_2 = 6$
Function value of criterion (01)	46
Function value of criterion (02)	0

Will the decision-maker be really satisfied knowing that the function value of the more important criterion is just equal to zero? Is it normal that the criterion (02) is executed at a lower degree than the criterion (01)? The relative distance between the final function value and its maximal value (obtained in the set of admissible solutions) is longer for the second criterion than for the first one, see Fig. 6.

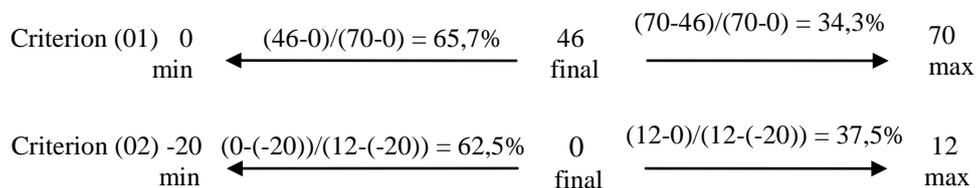


Fig. 6. The distances between final and optimal solutions

The method illustrated above is commonly known and presented in many manuals of economics (Sikora 2008; Trzaskalik 2008), but, as one may observe, it is not a rational procedure. The final solutions generated by this method do not actually take into consideration the criteria's significance declared at the beginning by the decision-maker.

Lecturers should also avoid problems which do not occur in the real world (Smoluk 2004). A topic which, in my opinion, may be questionable is the Closed Leontief Model (CLM)¹⁰. This model assumes that an economy consists of n interdependent industries (or sectors), where each industry consumes some of the goods produced by the other industries, including itself, and where no goods leave or enter the system. The presentation of such a problem is desirable provided that more complex and realistic topics (i.e. Open Leontief Model, Dynamic Leontief Model) are discussed as well. Meanwhile, the number of hours dedicated to Mathematics or Econometrics has recently decreased at Polish universities of economics, so the lecturer is just able to describe one extremely simplified model (CLM), which as a matter of fact can only be applied in very rare real decisive problems.

7) Spend much time on the interpretation of the results

The main problem occurs when the student obtains a result, but he/she does not know:

- whether such a solution is possible and logical (compare Żylicz 2009),
- what this result means.

Therefore, teachers ought to find enough time to explain (for instance)¹¹:

- why the coefficient of determination must not be negative or higher than 1,
- why the correlation coefficient must belong to the interval $[-1; 1]$,
- why the standard deviation is never negative (Maddala 2006),
- why the total activity float is always non-negative,
- why the minimum cut is determined by arcs connecting the labeled vertex with the unlabeled one.

¹⁰ This topic is discussed within the Econometrics courses and/or Mathematics courses.

¹¹ The first three examples concern the Econometrics, Statistics, Forecasting and Simulation courses. The fourth example is related to Operations Research (Time Project Network Analysis) and Project Management and the last one – to Operations Research or Operations Research in Logistics (Maximum Flow Problem).

These examples seem to be obvious for the lecturers, but not for the majority of students.

8) Remember that solutions obtained by a mathematician may not satisfy an economist

I always emphasize during my classes that not each optimal solution obtained for a given problem thanks to mathematical rules is equally good for an economist (Żylicz 2009). Let us analyze three cases:

A) A mathematician checks, first of all, whether a given econometric model is statistically correct, i.e. he verifies the goodness of “fit” (by means of the coefficient of determination) and the statistical significance of all parameters belonging to this model. Meantime, an economist must additionally verify its economic sense (e.g. may this variable be influenced by these factors?).

B) From the mathematical point of view all production plans which give the same highest revenue are similarly the best, i.e. optimal. However, an entrepreneur has always other supplementary hidden goals (e.g. to use as few resources as possible or to have a diverse product range). That is why, the economist’s set of optimal solutions is often less numerous than the set obtained by a mathematician.

The model presented below concerns the revenue maximization. The variables x_1 , x_2 signify the quantity of the product A and B and the constraints (1) and (2) show how the resource S1 and S2 should be exploited.

$$(0) 6 x_1 + 15 x_2 \rightarrow \max$$

$$(1) 6 x_1 + 4 x_2 \leq 40$$

$$(2) 2 x_1 + 5 x_2 = 20$$

$$(3) x_1, x_2 \in N$$

Mathematicians see two optimal solutions (Table 5), but an economist will not treat them similarly.

Table 5. Optimal solutions

Solutions	Objective function's value	Degree of resource exploitation (S1)	Number of different products
1) $x_1 = 0; x_2 = 4$	60	$16/40 = 40\%$	1
2) $x_1 = 5; x_2 = 2$	60	$38/40 = 95\%$	2

C) In the sensitivity (post-optimisation) analysis the parameter's interval for which a given optimal solution does not change, may contain, depending on the case considered, positive and negative values as well, but an economist will eliminate the last ones if this parameter signifies e.g. a product price, i.e. a coefficient that ought to be always non-negative.

9) Do not require students to learn numerous and complex formulas by heart for the test

When the list of equations is accessible during the test, students deceive more seldom. Furthermore, they may concentrate on understanding methods and on testing formulas at home and not on memorizing equations without comprehension.

However, I would like to emphasize that I am not a follower of making all formulas available to students. These which are very easy to derive should not be given. Showing equations for each linear (e.g. $y = ax + b$) and non-linear (e.g. $y = Ae^{bx}$) function neither does make sense¹².

10) If it is possible, the theoretical and practical parts of a given course should be carried out by the same person.

This guideline may seem to be controversial, but since I have the opportunity to present all the subject (lecture and classes) I have observed many changes for the better. Firstly, the teacher uses the same vocabulary and methods during the lecture and the classes, which facilitates the subject comprehension. Secondly, he has a better control of the order of the topics presented (theory before practice), which is extremely important especially when some hours cannot be led because of unexpected extra events or when a given topic requires more time than it was scheduled. The classes' teacher does not have to check whether a given topic has been explained during the lecture and does not have to present this problem if it has not been theoretically discussed before, because he perfectly knows what has been done. Thirdly, students more often attend the lectures since they do not want to make themselves unpopular with the person leading the classes and higher attendance usually means higher grades.

¹² This topic is discussed within the Econometrics courses and/or Mathematics courses.

3. Conclusions

In my opinion, if we fulfill the conditions presented above, then:

- the atmosphere in class will be more pleasant,
- economic quantitative subjects will be better understood by the students,
- the listeners will be more interested in topics presented by the lecturer,
- marks will be higher.

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