A TIME SERIES ANALYSIS OF AGGREGATE CONSUMPTION FUNCTION FOR PAKISTAN

Aggregate Consumption, being an important part of National Income Accounts, has been intensively researched in macroeconomics. Aggregate consumption and aggregate savings have powerful influence on an economy’s long-term productive capacity and capture the macroeconomic fluctuations and business cycles more appropriately. This interest reflects the belief that the structural forms of consumption functions and theories are a key to solve many business cycle issues. This study provides a time series analysis of aggregate consumption function for Pakistan by using a quarterly data from 1973(1) to 2010(4). The DHSY’s error correction and Hall’s random walk models are empirically tested. The data validate the application of both methodologies in estimating the aggregate consumption function. Hall’s martingale hypothesis also holds showing that current consumption is a good predictor of future consumption in Pakistan. In the context of Pakistan, 86% of the income has been consumed in the long run while the rest is saving. Inflation is unanticipated but is not accelerating. The data provide enough evidence to reject the price homogeneity hypothesis, however we are unable to reject the hypothesis of the unit elasticity of income. The Mankiw and Campbell test concludes that there are 49% of consumers who are backward looking while 51% follow permanent income hypothesis and are forward looking.

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1. INTRODUCTION

Aggregate consumption determines aggregate savings which entail a portion of national income which is not consumed. Aggregate consumption and aggregate savings have a powerful influence on the economy’s long term productivity capacity and capture the macroeconomic fluctuations and business cycles more appropriately. This amount of attention reflects the belief that the structural forms of consumption functions and theories are the key to solve many business cycle issues.

Consumption theories have evolved over the decades and different consumption functions are formulated. After the great depression of the
1930s, consumption theories became the focus for many economists and researchers. The most famous work was done by Keynes (1936), who developed a consumption function in the form of the Absolute Income Hypothesis (AIH). The consumption theories have radically changed after the 1950s with the emergence of the Permanent Income Hypothesis (PIH) presented by Milton Friedman (1957), and the Life Cycle Hypothesis (LCH) developed by Modigliani and Brumberg (1957). In 1989, Campbell and Deaton calculated the proportion of individuals who follow the Rule of Thumb (ROT), i.e. AIH and those who are forward looking and follow PIH.

After the plethora of work on these theories by many economists, Hall (1978) presented another theory of consumption which is known as the Random Walk Model. A major portion of the empirical literature on aggregate consumption is focused on Hall’s demonstration that current consumption depends only on its own lag. Hall is followed by many researchers who have tested the LCPI hypothesis under rational expectations such as Bilson (1980), Flavin (1981), Blinder and Deaton (1985) and Mankiw (1982). He also investigated that under some assumptions the PIH implies that consumption must be a martingale process, that is no variable other than current consumption helps in predicting future consumption. Haug (1991) proposed that empirical rejections of the LCPI model are due to the time aggregation bias. His empirical results support Hall’s hypothesis that consumption is random walk. The findings in Hayashi (1985) are in line with Hall and Mishkin (1982).

In 1978, Davidson, Hendry, Srba, and Yeo (DHSY) noticed that no consensus has emerged about the short-term dynamic interaction between disposable income and consumer expenditure in the United Kingdom, despite the superficial similarity of the published models. In contention to explain the diversity of the published estimates, DHSY incorporated the dynamic econometric models to estimate the consumption function. DHSY’s model was a revolution in the traditional consumption theories as it provided a more sophisticated technique to estimate consumption by incorporating all the previous consumption functions. All these theories and econometric models have been empirically evaluated by many economists in the context of developing and developed economies. The results vary from country to country.

This study investigates the application of DHSY’s dynamic model and Hall’s random walk model in the context of Pakistan by using quarterly aggregate consumption data from 1973(1) to 2010(4). We also investigate the ratio of consumers who follow ROT and those who follow PIH, using the
Campbell and Mankiw (1989) test. Furthermore, we tested the hypothesis of (i) unit elasticity, (ii) price homogeneity, and (iii) the martingale process in the context of Pakistan.

2. THE DATA & METHODOLOGY

The data used in this study comprises of the following variables:
\[
\ln C_t = c_t \quad \text{Aggregate Consumption Expenditure constant at 1999-2000 prices}
\]
\[
\ln Y_t = y_t \quad \text{Gross Domestic Product at Factor Cost constant at 1999-2000 prices}
\]
\[
P_t \quad \text{Inflation as an implicit function of Consumption}
\]

The data is collected on a quarterly basis from 1973 to 2010 and acquired from a report published by the State Bank of Pakistan on quarterization of national income accounts (Hanif, Iqbal, Malik, 2013). The variables estimated are in million Rupees.

2.1. Methodology

The DHSY Model

DHSY showed that the Error Correction Model (ECM) would yield a robust short-term (dynamic) relationship between Aggregate Consumption Expenditures and Personal Disposable Income. DHSY considered the steady state theory and presented a model which would generate a long-term unitary income elasticity of consumption.

DHSY use quarterly seasonally unadjusted data in their empirical analysis. The model to be considered here is:

\[
\Delta_4 c_t = \alpha_1 \Delta_4 y_t + \alpha_2 \Delta_1 \Delta_4 y_t + \alpha_3 \ln \left( \frac{C}{Y_{1-4}} \right) + \\
+ \alpha_4 \Delta_4 P_t + \alpha_5 \Delta_1 \Delta_4 P_t + \alpha_6 \Delta_4 c_{t-4} + \varepsilon_t
\]

the symbols \( \Delta \) are the difference operators.

Under the permanent income hypothesis, if \( \alpha_1 \) is positive then some of the increase in income is treated as an increase in permanent income. If \( \alpha_2 \) is positive then, if the consumer considers this extra rise to be transitory income, the consumer will reduce consumption from the level implied by
treating the whole income increase as permanent. The alternative argument would hold if $\alpha_2$ is negative. Thus, on economic theory grounds $\alpha_2$ may be expected to be negative.

The restriction of dropping the intercept was considered by DHSY because they retain an ECM term of the form $\ln \left( \frac{C}{Y} \right)_{t-4}$.

For static equilibrium, the equation is solved as:

$$\alpha_3 (c_{t-4} - y_{t-4}) = 0$$

and if $\alpha_3 \neq 0$ then $c_t = y_t$, which implies a long-term unitary income elasticity of consumption.

Inflation is often unanticipated with consumers interpreting the price rises of goods they typically purchase as relative price increases rather than as a general rise in the aggregate price level, which may be because inflation is accelerating. This response of consumers to unanticipated inflation viewed in this way would be to reduce their real consumption leading to what Deaton termed “involuntary saving through unanticipated inflation” (Deaton, 1977). Hence despite consumer expenditure and income variables being measured in real terms, DHSY did also find a role for inflation effects. According to Deaton’s argument, if inflation is unanticipated, $\alpha_4$ will be negative and if inflation is both accelerating and unanticipated then $\alpha_5$ will also be negative.

To test the hypothesis of price homogeneity ($\sum \beta_i = 0$) and unit elasticity of income ($\sum \alpha_i \approx 1 - \sum \gamma_i$) the following model is used.

$$c_i = \sum_{i=0}^{5} (\alpha_i y_{i-1} + \beta_i p_{i-1}) + \sum_{i=1}^{5} (\gamma_i c_{i-1}) + \epsilon_i.$$  \hspace{1cm} (2)

**Random Walk Model**

Hall (1978) combined the life cycle and permanent income models with the rational expectations to conclude that consumption follows the random walk, that is changes in consumption over time are unpredictable. The final equation of RWM with drift can be written in terms of a regression model as:

$$c_i = \beta_i c_{i-1} + \epsilon_i.$$  \hspace{1cm} (3)
The Random Walk hypothesis implies that there is no need for other variables for forecasting because all the information is already there in $C_{t-1}$ decision, thus adding other variables without predictive power means that they will be statistically insignificant (Hall, 1978).

**Martingale Hypothesis**

Another implication of Hall’s model is that consumption follows a martingale. It implies that changes in consumption should be uncorrelated with unanticipated changes in income, i.e.

$$E_{t-1}(\Delta_4 c_t) = 0.$$ 

To test this implication the following model is used which in fact is a special case of Hall’s 1978 test of the Martingale model for consumption.

$$\Delta_4 c_t = \alpha + \sum_{i=1}^{n} \beta_i \Delta_4 y_{t-i} + \epsilon_i$$

(4)

$$H_0 : \beta_1 = \beta_2 = \ldots = \beta_n = 0 \rightarrow c_t \text{ is a martingale}$$

**Campbell and Mankiw Model**

Campbell and Mankiw in 1990 separated the proportion of forward looking consumers from backward looking consumers. They assumed that a proportion ($\lambda$) of consumers follow the “Rule of Thumb (ROT)” and consume their current income (AIH) while proportion ($1-\lambda$) of individuals are forward looking and satisfy the Permanent Income Hypothesis (PIH).

For Rule of Thumb consumers:

$$\Delta_4 c_{t}^{\text{rot}} = \Delta_4 y_{t}^{\text{rot}}.$$  

For Permanent Income Hypothesis followers:

$$\Delta_4 c_{t}^{p} = \epsilon_t,$$

where $E_{t-1}\epsilon_t = 0$. Since $\Delta_4 c_t = \Delta_4 c_{t}^{\text{rot}} + \Delta_4 c_{t}^{p}$ and $\Delta_4 y_{t}^{\text{rot}} = \lambda \Delta_4 y_t$, we have

$$\Delta_4 c_t = \lambda \Delta_4 y_t + \epsilon_t.$$  

(5)

The error term is an innovation to consumption plus other possible errors. The model (5) cannot be estimated directly by OLS because $y_t$ may be
correlated with $\varepsilon_t$. The standard instrumental technique may be applied but we will be using the following alternative method. Assume,

$$\Delta_c t = \alpha_0 + \alpha_1 \Delta y_{t-1} + u_t , \quad (6)$$

If we estimate this model by OLS, we get $\hat{\alpha}$ which converges to $\alpha$ in large samples. Substituting equation (6) by equation (5), we get:

$$\Delta_c t = \lambda (\alpha_0 + \alpha_1 \Delta y_{t-1} + u_t) + \varepsilon_t , \quad (7)$$

where, $\beta_1 = \frac{\alpha t \lambda}{\alpha} \rightarrow \lambda = \beta_1 / \alpha$, and $\eta_t = \lambda u_t + \varepsilon_t$ has expectation equal to zero and it is uncorrelated with the lagged variables.

The instrumental method (IV) method breaks down if $\alpha$ is in the vicinity of zero (due to a weak instrument) particularly when we divide out by $\hat{\alpha}$ rather than $\alpha$, our results can be very noisy.

### 3. RESULTS DISCUSSION

#### 3.1. The DHSY Model (Error Correction Model)

The estimation result of final model of DHSY i.e. equation (1) is given below:

$$\Delta c_t = 0.37 \Delta y_t + 0.79 \Delta_4 y_t - 0.099 \ln \left( \frac{C}{Y} \right)_{t-4} - 0.046 \Delta_4 P_t + 0.079 \Delta_4 P_t + 0.57 \Delta_4 c_{t-1} .$$

All the variables are in log form and are seasonally differenced. The t-values are given in the parenthesis. The diagnostic tests show no autocorrelation, no heteroscedasticity and no model misspecification. According to the feedback theory, consumers plan to spend the same income in each year, modified by a proportion of their annual change in income ($\alpha_1 = 0.37$) and whether the change in income is positive or negative ($\alpha_2 = 0.79$). These together determine a short-term consumption decision which is altered by ($\alpha_3 = -0.099$) ensuring coherence with the long-term outcome $C_t = KY_t$. The impact elasticity is 1.158 ($0.37 + 0.79$) falling to 0.37 (37%) after one quarter. If the change in income is assumed to be
permanent, initially people would consume more than the change in income (entailing them to purchase durable goods or other things they have long planned to but could not afford because of income constraint) but the consumption will go down after one quarter to 37%. However, the long-term MPC is 0.86 \([0.37/(1-0.57)]\) which corroborates with the findings in Khan (1993).

The sign of Error Correction Term (ECM) is negative, which shows that in the long run APC is inversely related to the growth rates of consumption and income. Also, \(\alpha_3 \neq 0\) which confirms that \(C_t = Y_t\) and implies a long-term unitary income elasticity of consumption. The rate of adjustment is 10% which is very slow. If speed of adjustment is slow then APC is expected to decline (Figure 1).

![Fig. 1. APC](source: prepared by the authors)

Under Deaton’s argument, the negative sign of inflation coefficient shows that inflation is unanticipated and the positive sign of rate of inflation shows that it is not accelerating (Deaton, 1977). The response of consumers to unanticipated inflation would be to reduce real consumption leading to what Deaton has termed “involuntary saving through unanticipated inflation”.

Figure 2 shows that there is a good agreement between model forecasts and realizations. Both the chi-squared and Chow tests imply the parameter stability between the sample and post sample periods as all the projections are within the ±2 SD bounds. The RMSE is 0.0128 showing low forecast error.

![Figure 2. Eight Quarters forecast of the Error Correction Model](image)

Source: prepared by the authors

**The DHSY’s Unrestricted Model**

The estimation results for equation (2) are given below:

\[
c_t = 1.128 y_t - 0.84 y_{t-1} + 0.01 y_{t-2} - 0.037 y_{t-3} - 0.32 y_{t-4} + 0.247 y_{t-5} - 0.029 P_t - 0.133 P_{t-1} + 0.06 P_{t-2} + 0.05 P_{t-3} - 0.032 P_{t-4} + 0.07 P_{t-5} + 0.64 c_{t-1} - 0.02 c_{t-2} + 0.07 c_{t-3} + 0.26 c_{t-4} - 0.139 c_{t-5}.
\]

In this model all the variables are on levels and in log form. The diagnostic tests show no evidence of autocorrelation, heteroscedasticity and
model misspecification. The unrestricted form the DHSY’s error correction is estimated to test the following hypothesis:

1. **Price Homogeneity**

\[ H_0: \sum \beta_i = 0 \quad \rightarrow \text{Prices are homogeneous} \]
\[ H_1: \sum \beta_i \neq 0 \quad \rightarrow \text{Prices are heterogeneous} \]

2. **Unit Elasticity of Income**

\[ H_0: \sum \alpha_i + \sum \gamma_i = 1 \]
\[ H_1: \sum \alpha_i + \sum \gamma_i \neq 1 \]

The parameter restriction tests reveal that the hypothesis of homogeneous prices does not hold in the case of Pakistan (\(\chi^2 = 10.63 [0.001]\)). However, we fail to reject the unit elasticity of income (\(\chi^2 = 2.26 [0.132]\)). It also confirms that \(C_t = Y_t\) which implies long-term unitary income elasticity of consumption.

**Random Walk Model**

The estimation of the final equation of the Random Walk Model (RMW) is:

\[
c_t = 1.0009 \frac{c_{t-1}}{(0.0009)},
\]

\[ H_0: \beta_1 = 1 \quad \chi^2(1) = 0.99005 [0.3197]. \]

The data do not provide enough evidence to reject the null hypothesis. It proves the validity of Hall’s random walk model in the case of Pakistan. The results and test statistics are significant.

The forecast shows that all the values of projection are within the 95% forecast bounds. The parameter constancy test (Chow \(F(8,142) = 1.4673 [0.1743]\)) imply that the null hypothesis cannot be rejected i.e. no parameter value changes between the sample and post sample periods. The forecast accuracy is validated by the chi-square test (Forecast \(\chi^2(8) = 11.738 [0.1633]\)).
The Martingale Hypothesis

Regressing consumption on five lags of income by OLS (eq. 4), we get following estimated equation:
\[ \Delta c_t = 0.024 + 0.19 \Delta y_{t-1} + 0.26 \Delta y_{t-2} + 0.10 \Delta y_{t-3} - \\
0.196 \Delta y_{t-4} + 0.126 \Delta y_{t-5} \]

\( t \)-statistic: (2.23), (1.16), (1.53), (0.60), (-1.18), (0.80).

H\(_0\): \( \beta_1 = \beta_2 = \ldots = \beta_5 = 0 \rightarrow C_t \) is a martingale,
H\(_1\): at least one \( \beta_i \neq 0 \rightarrow C_t \) is not a martingale.

The F-test does not allow to reject the null hypothesis (F(5,137) = 1.8027 [0.1163]). This implies that consumption follows a martingale process, i.e. no variable other than current consumption can help in predicting future consumption.

Campbell and Mankiw Model

The Campbell and Mankiw test helps us to determine the proportion of individuals who follow the rule of thumb (ROT). Equation 6 and 7 are
estimated and solved for the parameter $\lambda$, the proportion of those who follow ROT.

$$\Delta c_t = 0.03 + 0.39 \Delta y_t \quad (t\text{-statistic})$$

$$\Delta c_t = 0.03 + 0.19 \Delta y_{t-1} \quad (t\text{-statistic})$$

$$\lambda = \frac{\beta}{\alpha} = \frac{0.19}{0.39} = 0.49$$

This implies that 49% of individuals follow ROT and 51% follow PIH. Those consumers who follow ROT, i.e. Keynesian absolute income hypothesis, believe that current consumption depends on current income and consumption changes with the change in income. However, the PIH followers respond to changes in their permanent income. Hence the temporary changes in their income (associated to business cycles) have little effect on their consumption. They utilize their savings to smooth their consumption pattern.

**CONCLUSION**

Aggregate Consumption, being an important part of National Income Accounts, has been intensively researched in macroeconomics. Aggregate consumption and aggregate savings have a powerful influence on the economy’s long term productive capacity and capture the macroeconomic fluctuations and business cycles more appropriately. This attention reflects the belief that the structural forms of consumption functions and theories are the key to solve many business cycle issues.

This study empirically proves the validity of Hall’s random walk model and DHSY’s error correction model in the context of Pakistan by using quarterly data of consumption from 1973(1) to 2010(4). Both models provide a stable relationship between consumption and income but DHSY’s model has a slight edge over Hall’s model in terms of forecast quality. Hall’s martingale hypothesis also holds which shows that in Pakistan current consumption is a good predictor of future consumption.

In the context of Pakistan, 86% of the income has been consumed in the long run while rest is saving. Inflation is unanticipated but not accelerating. The data provide enough evidence to reject the price homogeneity hypothesis, however we are unable to reject the hypothesis of unit elasticity.
of income. The Mankiw and Campbell test concludes that there are 49% consumers who still follow ROT and are backward looking while 51% follow PIH and are forward looking.

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