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ON THE ONSET OF THERMAL CONVECTION IN A LAYER OF OLDROYDIAN VISCO-ELASTIC FLUID SATURATED BY BRINKMAN–DARCY POROUS MEDIUM

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Abstract: Thermal instability in a horizontal layer of Oldroydian visco-elastic fluid in a porous medium is investigated. For porous medium the Brinkman–Darcy model is considered. A linear stability analysis based upon perturbation method and normal mode technique is used to find solution of the fluid layer confined between two free-free boundaries. The onset criterion for stationary and oscillatory convection is derived analytically. The influence of the Brinkman–Darcy, Prandtl–Darcy number, stress relaxation parameter on the stationary and oscillatory convection is studied both analytically and graphically. The sufficient condition for the validity of PES has also been derived.

Key words: *thermal instability, Oldroydian visco-elastic fluid, Brinkman–Darcy number, Prandtl–Darcy number*

1. INTRODUCTION

The onset of thermal convection in a horizontal layer of fluid saturated by a porous medium is regarded as a classical problem due to its wide range of applications in geophysics, agricultural product storage, enhanced oil recovery, packed-bed catalytic reactors and the pollutant transport in underground. Many researchers have investigated thermal instability problems by taking different types of fluids. A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [1]. Lapwood [2] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [3]. McDonnell [4] suggests the importance of porosity in the astrophysical context. A good account of convection problems in a porous medium is given by Vafai and Hadim [5], Ingham and Pop [6] and Nield and Bejan [7]. It is found that the acceleration term in the Darcy equation is not commonly used as the value of the Prandtl–Darcy number may vary between 10^{-2}–10^{23} depending on the nature of the porous medium. For traditional porous media applications, this value may be very large, thus providing justification for neglecting the acceleration term in the Darcy equation. However, there are some modern porous media applications, such as the situation involving fractured porous media, in which the value of the Vadasz number is of order unity or even smaller, thereby justifying the inclusion of the acceleration term. Thermal instability in Brinkman porous medium has been studied by Kuznetsov and Nield [8], Chand Rana [9] for nanofluids and found that Brinkman porous medium plays an important role in instability of fluid layer.

The above literature deals with the study of fluids as Newtonian fluids. In technological fields there exists an important class of fluids, called non-Newtonian fluids, which are also being studied extensively because of their practical applications, such as fluid film lubrication, analysis of polymers in chemical engineering, etc. An experimental demonstration by Toms et al. [10] has revealed that a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of Oldroyd [11]. The problem of convective instability of visco-elastic fluid heated from below was first studied by Green [12]. Vest and Arpaci [13] have investigated the problem of overstability in a horizontal layer of fluid. Bhatia and Steiner [14] have studied the problem of thermal instability of Oldroydian visco-elastic fluids. Sharma [15] has studied the effect of rotation on thermal instability of a visco-elastic fluid in a nonporous medium. Sharma and Kumar [16], Prakash and Chand [17] studied the thermal instability of an Oldroydian visco-elastic fluid in a porous medium. Recently, Chand [18], [19], Chand and Kango [20], Chand and Rana [21], Thakur and Rana [22], Chand and Rana [23] studied the thermal instability of various types of elastic viscous fluids in a porous medium.

In the present paper, an attempt has been made to study the thermal instability of Oldroydian visco-elastic fluid in a Brinkman–Darcy porous medium.

2. FORMULATION OF PROBLEM

Consider an infinite horizontal layer of Oldroydian visco-elastic fluid of thickness "d" bounded by plane $z = 0$ and $z = d$ in porous medium of porosity ε and medium permeability k_1. Fluid layer is acted upon by gravity force g (0, 0, g). The layer is heated from below and surfaces $z = 0$ and $z = d$ are maintained at constant T_0 and T_1 ($T_0 > T_1$), so that uniform temperature gradient is maintained.

The fluids described by the constitutive relations

$$p'_{ik} = p_{ik} - \delta_{ij} p'',$$

$$\left(1+\lambda\frac{d}{dt}\right)p_{ik} = 2\mu\left(1+\lambda_0\frac{d}{dt}\right)e_{ik},$$

$$e_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_j}\right),$$

where $p'_{ik}, p_{ik}, e_{ik}, \delta_{ik}$ and p'' denote respectively the normal stress tensor, shear stress tensor, rate of strain tensor, Kronecker delta and scalar pressure. Here, $\frac{d}{dt}$ is the convection derivative, λ the relaxation time and ($\lambda_0 < \lambda$) is the retardation time. If $\lambda_0 = 0$ the fluid is Maxwellian visco-elastic; while $\lambda_0 \neq 0$, then fluid is referred to as Oldroydian visco-elastic fluid and $\lambda = \lambda_0 = 0$, then fluid is known as Newtonian viscous fluid.

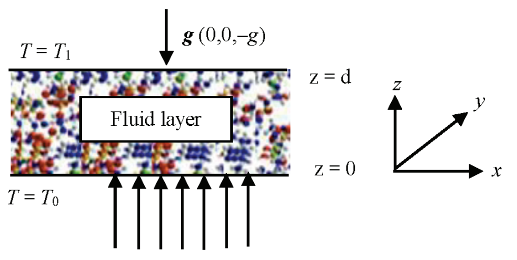

Fig. 1. Physical configuration of the problem

Let p, $\boldsymbol{q}$ (u, v, w), ρ, T, α, μ, $\tilde{\mu}$, and κ be the pressure, Darcy velocity, density, temperature, thermal coefficient of expansion, viscosity, effective viscosity and thermal diffusivity fluid, respectively.

The equation of motion, continuity and heat conduction for Oldroydian visco-elastic fluid in the Brinkman porous medium are

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\rho}{\varepsilon}\frac{d\boldsymbol{q}}{dt} = \left(1+\lambda\frac{\partial}{\partial t}\right)(-\nabla p + \rho\boldsymbol{g}) - \left(1+\lambda_0\frac{\partial}{\partial t}\right)\left(\tilde{\mu}\nabla^2 \frac{\mu}{k_1}\right)\boldsymbol{q}, \quad (1)$$

$$\nabla\cdot\boldsymbol{q} = 0, \quad (2)$$

$$E\frac{\partial T}{\partial t} + (\boldsymbol{q}\cdot\nabla)T = \kappa\nabla^2 T, \quad (3)$$

where $E = \varepsilon + (1+\varepsilon)\frac{\rho_s C_s}{\rho_0 C}$ and $\rho_0, C; \rho_s, C_s$ stand for density and heat capacity of fluid and solid matrix, respectively, and $\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\boldsymbol{q}\cdot\nabla)$ stands for convection derivative.

The equation of state is

$$\rho = \rho_0[1-\alpha(T-T_0)], \quad (4)$$

where the suffix zero refers to values at reference level $z = 0$, i.e., ρ_0, T_0 stands for density, temperature at lower boundary $z = 0$.

We assume that temperature is constant at the boundaries, so boundary conditions are

$$w = 0,\ T = T_0 \ \text{at}\ z = 0 \ \text{and}\ w = 0,\ T = T_1 \ \text{at}\ z = d. \quad (5)$$

The steady state solution is

$$q = (0,0,0), \quad T = T_0 - \frac{\Delta T}{d}z,$$

$$\rho = \rho_0\left(1+\alpha\frac{\Delta T}{d}z\right), \quad p_s = p_0 - \rho_0 g\left(z + \alpha\frac{\Delta T}{2d}z^2\right).$$

3. PERTURBATION EQUATIONS

Let q' (u', v', w'), T', p' be the perturbation in Darcy velocity $\boldsymbol{q}$ (initially zero), temperature T and pressure p, respectively. Substituting these in equations (1)–(3) and neglecting higher order terms of the perturbed quantities, we get the linearized perturbation equations as

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\rho_0}{\varepsilon}\frac{d\boldsymbol{q}'}{dt}=\left(1+\lambda\frac{\partial}{\partial t}\right)[-\nabla p'+\rho_0\alpha T'g]$$
$$-\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left(\tilde{\mu}\nabla^2-\frac{\mu}{k_1}\right)\boldsymbol{q}',\qquad(6)$$

$$\nabla\cdot\boldsymbol{q}'=0,\qquad(7)$$

$$E\frac{\partial T'}{\partial t}-w'\frac{\Delta T}{d}=\kappa\nabla^2T'.\qquad(8)$$

Introducing non-dimensional variables as

$$(x'',y'',z'')=\left(\frac{x',y',z'}{d}\right),$$

$$(u'',v'',w'')=\left(\frac{u',v',w'}{\kappa}\right)d,$$

$$t''=\frac{\kappa}{d^2}t,\quad p''=\frac{k_1}{\mu\kappa}p',\quad T''=\frac{T'}{\Delta T}.$$

[Dashes ($''$) are suppressed for convenience]

Equations (6)–(8) in non-dimensional form can be written as

$$\frac{1}{Va}\left(1+F\frac{\partial}{\partial t}\right)\frac{\partial q}{\partial t}=\left(1+F\frac{\partial}{\partial t}\right)(-\nabla p+RaT\hat{e}_z)]$$
$$+\left(1+F_0\frac{\partial}{\partial t}\right)(\tilde{D}a\nabla^2\boldsymbol{q}-\boldsymbol{q}),\qquad(9)$$

$$\nabla\cdot\boldsymbol{q}=0,\qquad(10)$$

$$E\frac{\partial T}{\partial t}-w=\nabla^2T,\qquad(11)$$

where the non-dimensional parameters are given as

$Pr=\dfrac{\mu}{\rho_0\kappa}$ is the Prandtl number,

$Da=\dfrac{k_1}{d^2}$ is the Darcy number,

$\tilde{D}a=\dfrac{\tilde{\mu}k_1}{\mu d^2}$ is the Brinkman–Darcy number,

$Va=\dfrac{\varepsilon Pr}{Da}$ is the Prandtl–Darcy number,

$F=\dfrac{\lambda\kappa}{d^2}$ is the stress relaxation parameter,

$F_0=\dfrac{\lambda_0\kappa}{d^2}$ is the strain retardation parameter,

$Ra=\dfrac{\rho_0\alpha gk_1\Delta Td}{\nu\kappa}$ is the Rayleigh number,

e_z is the unit vector along z-axis.

Operating equation (9) with e_z curl curl to eliminate pressure term p, we have

$$\frac{1}{Va}\left(1+F\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\nabla^2w=\left(1+F\frac{\partial}{\partial t}\right)(Ra\nabla_H^2T)$$
$$+\left(1+F_0\frac{\partial}{\partial t}\right)(\tilde{D}a\nabla^4w-\nabla^2w),\qquad(12)$$

where ∇_H^2 is two-dimensional Laplacian operator.

The boundary conditions are

$$w=0,\quad T=0\quad\text{at}\quad z=0$$

$$\text{and}\quad w=0,\quad T=1\quad\text{at}\quad z=1.\qquad(13)$$

4. NORMAL MODES AND STABILITY ANALYSIS

Analyzing the disturbances into the normal modes and assuming that the perturbed quantities are of the form

$$[w,T]=[W(z),\Theta(z)]\exp(ik_xx+ik_yy+nt),\qquad(14)$$

where k_x, k_y are wave numbers in x and y direction and n is the growth rate of disturbances.

Using equation (14), equations (11) and (12) become

$$\left[\left(\frac{1}{Va}(1+Fn)n(D^2-a^2)-(1+F_0n)(\tilde{D}a(D^2-a^2)^2-(D^2-a^2))\right)D^2\right]$$
$$\times W-a^2Ra\Theta=0,\qquad(15)$$

$$W+(D^2-a^2-nE)\Theta=0,\qquad(16)$$

where $D\equiv\dfrac{d}{dz}$ and $a=\sqrt{k_x^2+k_y^2}$ is the dimensionless resultant wave number.

Eliminating Θ between equations (15) and (16), we get

$$\left[(D^2-a^2-nE)\left[\left(\frac{1}{Va}(1+Fn)n(D^2-a^2)-(1+F_0n)(\widetilde{D}a(D^2-a^2)^2-(D^2-a^2))\right)\right]-a^2Ra\right]W=0. \qquad (17)$$

The boundary conditions of the problem in view of normal mode analysis are

$$W=0,\quad D^2W=0 \quad \text{at} \quad z=0 \text{ and } \quad z=1. \qquad (18)$$

The solution of equation (19) characterizing the lowest mode is

$$W=W_0\ \sin\pi z, \qquad (19)$$

which satisfy boundary conditions (18), where W_0 is constant.

Substituting solution (19) into equations (17), we obtain dispersion relation

$$Ra=\frac{(J^2+nE)\left(\frac{1}{Va}(1+Fn)nJ^2+(1+F_0n)(\widetilde{D}aJ^4+J^2)\right)}{a^2}, \qquad (20)$$

where $J^2=\pi^2+a^2$.

For neutral instability $n = i\omega$, (where ω is real and dimensionless frequency of oscillation) equation (20) reduces to

$$Ra=\Delta_1+i\omega\Delta_2 \qquad (21)$$

where

$$\Delta_1=\frac{J^2}{a^2}\left(\frac{J^2(1+\omega^2FF_0)(\widetilde{D}aJ^2+1)+\omega^2E(F-F_0)(\widetilde{D}aJ^2+1)}{1+\omega^2F^2}-\frac{\omega^2E}{Va}\right), \qquad (22)$$

and

$$\Delta_2=\frac{J^2}{a^2}\left(\frac{E(1+\omega^2FF_0)(\widetilde{D}aJ^2+1)-J^2E(F-F_0)(\widetilde{D}aJ^2+1)}{1+\omega^2F^2}-\frac{J^2}{Va}\right). \qquad (23)$$

Since Ra is a physical quantity, so it must be real. Hence, it follows from equation (21) that either $\omega = 0$ (exchange of stability, steady state) or $\Delta_2 = 0$ ($\omega \neq 0$ overstability or oscillatory onset).

5. STATIONARY CONVECTION

For stationary convection $n = 0$, equation (20) reduces to

$$(Ra)_s=\frac{(\pi^2+a^2)^2}{a^2}[(\widetilde{D}a(\pi^2+a^2)+1)]. \qquad (24)$$

We find that for the stationary convection, the stress relaxation time parameter F and strain retardation time parameter F_0 vanishes with n and hence the Oldroydian visco-elastic fluid behaves like an ordinary Newtonian fluid.

The critical value of the wave number on the onset of instability which is obtained from the condition $\left(\frac{\partial Ra}{\partial a}\right)_{a=a_c}=0$, depends upon the Brinkman–Darcy number.

If $\widetilde{D}a = 0$ the critical value of the wave number is attained at $a_c = \pi$, and critical value of the Rayleigh number is $(Ra)_c = 4\pi^2$ which is the classical result obtained by Lapwood [2] for Newtonian fluid.

If $\widetilde{D}a$ is very large as compared to the unity, then critical value of wave number is attained at $a = \pi/\sqrt{2}$, critical value of the Rayleigh–Darcy number is $(Ra)_c=\frac{27\pi^4}{4}$. This is exactly the same result of Rayleigh–Bénard instability for Newtonian fluid as obtained by Chandrasekhar [1].

Numerical computations are carried out for different values of the stress relaxation time parameter F, strain retardation time parameter F_0, Brinkman–Darcy $\tilde{D}a$ number and Vadaz Va. The parameters considered are in the range (Yang et al. [23], Chand and Rana [24]) of $10 \le Va \le 10^2$ (Vadaz number), $10^2 \le Ra \le 10^5$ (thermal Rayleigh number), $10^{-3} \le \widetilde{D}a \le 10^{-1}$ (Brinkman–Darcy number), $10^{-1} \le F \le 10^1$ (stress relaxation time parameter) and $10^{-1} \le F_0 \le 10^1$ (strain retardation time parameter).

To study the effect of Brinkman–Darcy number on the stationary convection, we examine the nature of $\frac{dRa}{d\widetilde{D}a}$ analytically.

Equation (24) gives $\frac{dRa}{d\widetilde{D}a}>0$, thus Brinkman–Darcy number stabilizes the fluid layer.

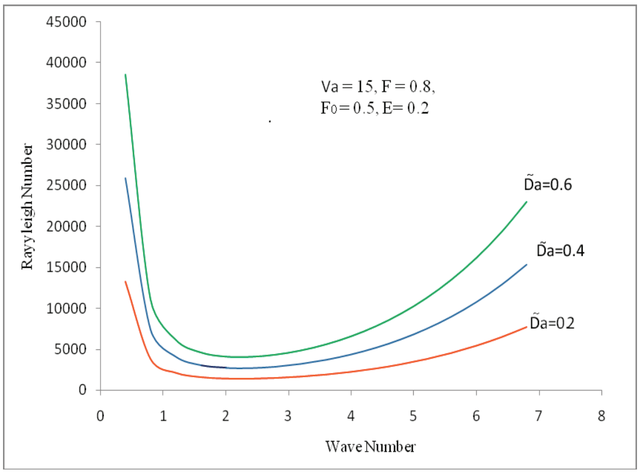

Fig. 2. Variation of the Rayleigh–Darcy number with wave number for different values of the Brinkman–Darcy number

Figure 2 shows the variation of stationary Rayleigh–Darcy number with the wave number for different values of the Brinkman–Darcy number and it is found that the Rayleigh–Darcy number increases with an increase in the value of Brinkman–Darcy number, thus the Brinkman–Darcy number has stabilizing effect on the stationary convection. This is in good agreement with the result obtained by Bala and Chand [25].

6. OSCILLATORY CONVECTION

For oscillatory convection $\omega \neq 0$, we must have $\Delta_2 = 0$, which gives

$$\omega^2 = \frac{J^2(F-F_0)(\tilde{D}aJ^2+1) - J^2\left(E(\tilde{D}aJ^2+1)+\dfrac{1}{Va}\right)}{EFF_0(\tilde{D}aJ^2+1)+\dfrac{J^2F}{Va}}.$$

If $(F-F_0)(\tilde{D}aJ^2+1) < J^2\left(E(\tilde{D}aJ^2+1)+\dfrac{1}{Va}\right)$, the oscillatory convection is not possible. Thus $(F-F_0)(\tilde{D}aJ^2+1) < J^2\left(E(\tilde{D}aJ^2+1)+\dfrac{1}{Va}\right)$ is a sufficient condition for the non-existence of oscillatory convection, the violation of which does not necessarily imply the occurrence of oscillatory convection.

Equation (21), with $\Delta_2 = 0$, oscillatory Rayleigh–Darcy number is given as

$$(Ra)_{osc} = \frac{J^2}{a^2} \cdot \left(\frac{J^2(1+\omega^2 FF_0)(\tilde{D}aJ^2+1)+\omega^2 E(F-F_0)(\tilde{D}aJ^2+1)}{1+\omega^2F^2} - \frac{\omega^2 E}{Va}\right). \quad (25)$$

To find the oscillatory solution, we first find the roots for ω^2 from equation (25). If there are no positive roots of ω^2, then oscillatory convection is not possible. If there are positive roots of ω^2, then oscillatory Rayleigh–Darcy number is obtained from equation (25) after substituting the positive value of ω^2.

Figure 3 shows the variation of oscillatory Rayleigh–Darcy number with wave number for different values of the Prandtl–Darcy number and it has been found that the Rayleigh–Darcy number increases with increase in the value of Prandtl–Darcy number, thus the Prandtl–Darcy number has stabilizing effect on oscillating convection.

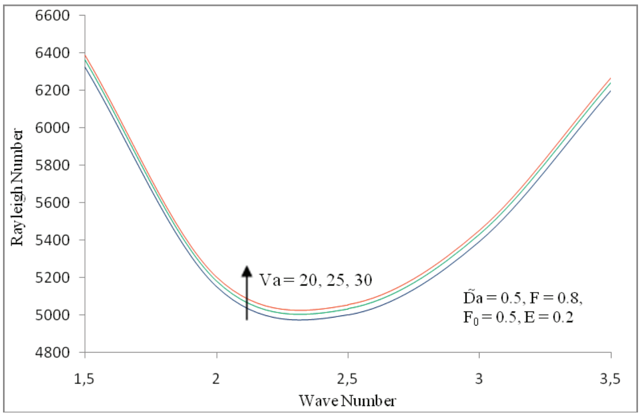

Fig. 3. Variation of the Rayleigh–Darcy number with wave number for different values of the Prandtl–Darcy number

7. PRINCIPLE OF EXCHANGE OF STABILITIES

Multiplying equation (15) by W^* and integrating the resulting equation over the vertical range of z, we get

$$\left(\frac{(1+F_0 n)}{(1+Fn)}\right)\int_0^1 \tilde{D}a(|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^4)dz + \left(\frac{(1+F_0 n)}{(1+Fn)}\right)\int_0^1 (|DW|^2 + a^2|W|^2)dz + \frac{n}{Va}\int_0^1 (|DW|^2 + a^2|W|^2)dz = -a^2 Ra\int_0^1 W^*\Theta dz. \quad (26)$$

Using equation (16), equation (26) becomes

$$\left(\frac{(1+F_0 n)}{(1+Fn)}\right)\int_0^1 \widetilde{D}a(|D^2W|^2+2a^2|DW|^2+a^4|W|^4)dz+\left(\frac{(1+F_0 n)}{(1+Fn)}\right)\int_0^1(|DW|^2+a^2|W|^2)dz$$
$$+\frac{n}{Va}\int_0^1(|DW|^2+a^2|W|^2)dz=a^2Ra\int_0^1(|D\Theta|^2+(a^2+n^*E)|\Theta|^2)dz. \tag{27}$$

Putting $n = n_r + in_i$ and equating imaginary part, we get

$$n_i\left\langle \begin{array}{l}\left(\frac{(F-F_0)}{(1+Fn_r)^2+n_i^2}\right)\int_0^1 \widetilde{D}a(|D^2W|^2+2a^2|DW|^2+a^4|W|^4)dz \\ +\left(\frac{(F-F_0)}{(1+Fn_r)^2+n_i^2}-\frac{1}{Va}\right)\int_0^1(|DW|^2+a^2|W|^2)dz-a^2RaE\int_0^1(|\Theta|^2)dz\end{array}\right\rangle=0. \tag{28}$$

Further since $W(0) = W(1)$, we have the Rayleigh–Ritz inequality,

$$\int_0^1(|DW|^2+a^2|W|^2)dz \ge \int_0^1(\pi^2+a^2)|W|^2dz\,. \tag{29}$$

Using equation (29), equation (28) becomes

$$n_i\left\langle \begin{array}{l}\left(\frac{(F-F_0)}{(1+Fn_r)^2+n_i^2}\right)\int_0^1 \widetilde{D}a(|D^2W|^2+2a^2|DW|^2+a^4|W|^4)dz \\ +\left(\frac{(F-F_0)}{(1+Fn_r)^2+n_i^2}-\frac{1}{Va}\right)(\pi^2+a^2)\int_0^1|W|^2dz-a^2RaE\int_0^1(|\Theta|^2)dz\end{array}\right\rangle=0\,. \tag{30}$$

Multiplying equation (16) with Θ^* and integrating the resulting equation over the vertical range of z, we get

$$\pi^4\int_0^1|\Theta|^2dz \le \int_0^1|W|^2dz\,. \tag{31}$$

Using equations (29), equation (30) gives

$$n_i\left\langle \begin{array}{l}\left(\frac{(F-F_0)}{(1+Fn_r)^2+n_i^2}\right)\int_0^1 \widetilde{D}a(|D^2W|^2+2a^2|DW|^2+a^4|W|^4)dz \\ +\left(\frac{(F-F_0)}{(1+Fn_r)^2+n_i^2}-\frac{1}{Va}\right)(\pi^2+a^2)\int_0^1|W|^2dz-\frac{R_a a^2E}{\pi^4}\int_0^1(|W|^2)dz\end{array}\right\rangle<0\,. \tag{32}$$

The term inside the brackets of equation (32) is positive definite if

$$RaE<\frac{\pi^4(\pi^2+a^2)}{a^2}\left(\frac{(F-F_0)}{(1+Fn_r)^2+n_i^2}-\frac{1}{Va}\right). \tag{33}$$

If $RaE<\frac{\pi^4(\pi^2+a^2)}{a^2}\left(\frac{(F-F_0)}{(1+Fn_r)^2+n_i^2}-\frac{1}{Va}\right)$, then n_i must be zero and PES valid.

8. RESULTS AND DISCUSSION

Thermal instability in a horizontal layer of Oldroydian visco-elastic fluid in a Brinkman porous medium is investigated. It should be noted that for the oscillatory convection it depends upon the stress relaxation time parameter F, strain retardation time parameter F_0, Brinkman–Darcy $\tilde{Da}$ number and Prandtl –Darcy (Vadaz number Va).

We discussed the results graphically.

Figure 4 shows the effect of the Brinkman–Darcy number on the oscillatory Rayleigh number. For each value of the Brinkman–Darcy number, it was found that a critical value of strain retatadation parameter F_0 exists which divides the boundary of regimes bewteen oscillatory and stationary convection. Initially convection begins in the oscillatory mode. As the value of F_0 reaches to the critical value, convection ceases to oscilltory and stationary convection occur. Also the increasing values of the Rayleigh–Darcy number with increases in the Brinkman–Darcy number, indicate stabilizing effect of the Brinkman–Darcy number on oscillating convection.

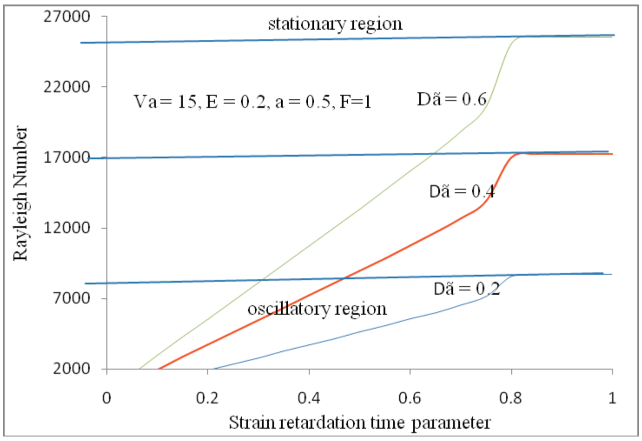

Fig. 4. Variation of the Rayleigh–Darcy number with strain retardation time parameter for different values of the Brinkman–Darcy number

Figure 5 shows the effect of the stress relaxation parameter on the oscillatory Rayleigh number. For each value of the stress relaxation parameter, it was found that a critical value of strain retardation parameter F_0 exists which divides the boundary of regimes between oscillatory and stationary convection. Also the values of the Rayleigh–Darcy number decrease with increases in the stress relaxation time parameter, thus stress relaxation time parameter destabilzes the oscillating convection.

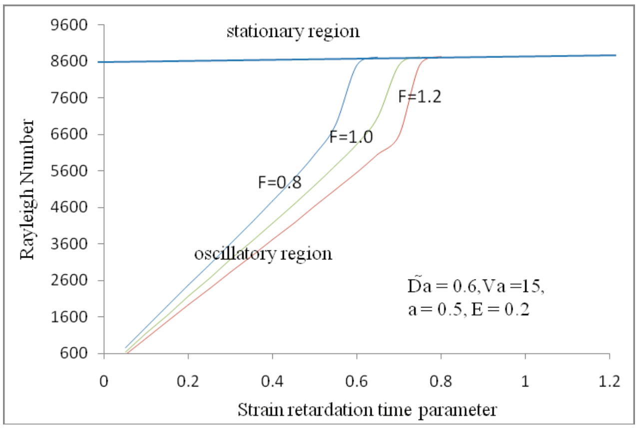

Fig. 5. Variation of the Rayleigh–Darcy number with strain time retardation parameter for different values of the stress relaxation time parameter

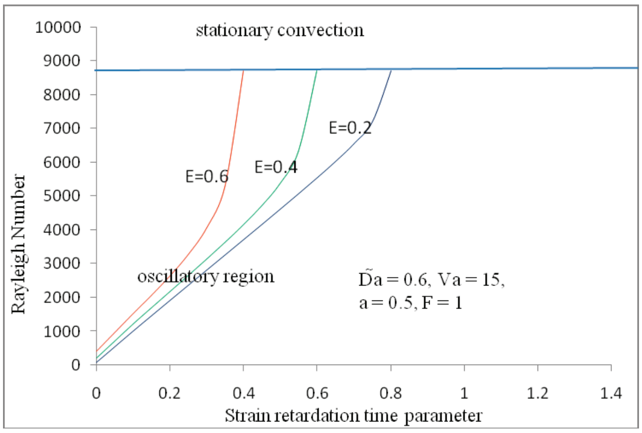

Fig. 6. Variation of the Rayleigh–Darcy number with strain retardation parameter for different values of density ratio

Figure 6 shows the effect of the density ratio on the oscillatory Rayleigh–Darcy number. The values of the Rayleigh–Darcy number increase with increases in the values of density ratio, thus denisty ratio has stabilzing effect on oscillating convection.

9. CONCLUSIONS

A linear analysis of thermal instability in a horizontal layer of Oldroydian visco-elastic fluid in the presence of Brinkman porous medium is investigated. Expressions for the Rayleigh–Darcy number, for the stationary convection and oscillatory convection have been obtained. The main conclusions of the present study are:

(i) In the stationary convection, the Oldroydian visco-elastic fluid behaves like an ordinary Newtonian fluid.

(ii) The Brinkman–Darcy number stabilizes the stationary convection.
(iii) The Prandtl–Darcy number and density ratio stabilizes oscillatory convection.
(iv) Stress relaxation time parameter destabilzes the oscillating convection.
(v) $RaE < \frac{(\pi^2 + a^2)}{a^2}\left(\frac{(F - F_0)}{(1+Fn_r)^2 + n_i^2} - \frac{1}{Va}\right)$ is the sufficient condition for the validity of principle of exchange of stabilities.

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