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Analysis and simulation of a method of measuring distributed birefringence in a polarization-maintained fiber

XIAOBIN XU*, ZHIHAO ZHANG, MINGHUA AN, FEI TENG, ZUCHEN ZHANG

Department of Opto-electronic Engineering, Beihang University,
Xueyuan Road, Beijing 100083, PR China

*Corresponding author: xuxiaobin@buaa.edu.cn

Based on a modified polarization-optical time-domain reflectometer technique, a tunable laser is applied to modulate reflection light intensity in a frequency domain, and the accurate distributed birefringence can be determined through the correlation analysis and the least square analysis of the reflected light intensity obtained by optical time-domain reflectometer. The simulation results indicate that the error of measurement is about one order less than the true value without consideration of polarization crossover, and two orders less than the true value with polarization crossover being considered.

Keywords: polarization-maintained fiber, birefringence, distributed measurement.

1. Introduction

Polarization-maintained (PM) fiber is generally a highly birefringent device, and axially non-homogeneous stress causes birefringence to distribute non-homogeneously, which affects severely its application in some occasions. For example, in a fiber optic gyroscope (FOG), randomly non-reciprocal birefringence in a fiber coil is a source of a bias error and noise, and it restricts the precision and long-time stability of FOG as it varies with time and environment. Therefore, measuring accurately distributed birefringence along PM fiber is necessary and significant.

In a normal single-mode (SM) fiber, inherent birefringence is very weak, and beatlength is generally from a few to tens of meters, so both the average and distributed birefringence can be measured easily with many methods, such as optical frequency-domain reflectometry (OFDR) [1, 2], polarization-optical time-domain reflectometer (P-OTDR) [3], method of equivalent beatlength [4]. However, in PM fiber, whether stress pattern or shape pattern, inherent birefringence is so high that its beatlength is on the order of millimeter. Although it allows determination of mean birefringence with the method of P-OTDR [3], distributed birefringence is difficult to obtain due to its short beatlength and limited resolution of OTDR. Reference [5] applies transient Brillouin grating to realize measurement, but it is a very complex system. The paper proposes a simple method to measure the distributed birefringence in PM fiber based on the modified P-OTDR technique [3], and it enables to resolve the conflict between a limited resolution of OTDR and short beatlength of PM fiber.

2. Principle of measurement

In general, an optical fiber with axially varying birefringence can be represented by a series of concatenated homogeneous elements [3, 6, 7], as illustrated in Fig. 1, where L_i is the length of i-th section. To acquire each section's birefringence, the principle of measurement is illustrated in Fig. 2. The principle is based on the setup of P-OTDR, but a tunable laser is applied to continuously modulate wavelength of the input light. OTDR is used to control the time sequence of tunable laser's pulse and record light intensity reflected from different locations along the fiber. The polarizer has a certain angle's (*e.g.*, 30°) alignment with PM fiber.

Optical wave of laser can be expressed as $E_{\text{in}} = [1, 0]^T$, and it is supposed to be on the passing axis of the polarizer, so reflected optical wave from n-th element is [7, 8]

$$E_{\text{out}}(z) = P T_{n-1}^T \left(R_n^T M_n^T R M_n R_n \right) T_{n-1} E_{\text{in}} \tag{1}$$

where polarizer's Jones matrix is $P = [1, 0; 0, 0]^T$, $M_n = [\exp(-j \Delta\beta_n L_n), 0; 0, 1]^T$, $R_n = [\cos(\theta_n), \sin(\theta_n); -\sin(\theta_n), \cos(\theta_n)]^T$, $\Delta\beta_n$ and L_n are birefringence and length of n-th element, respectively, θ_n is the angle between two adjacent elements, reflection

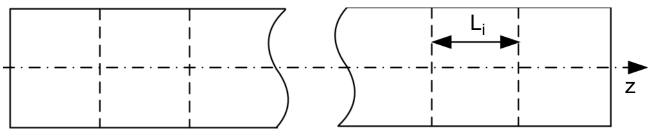

Fig. 1. Concatenated model of birefringence in PM fiber.

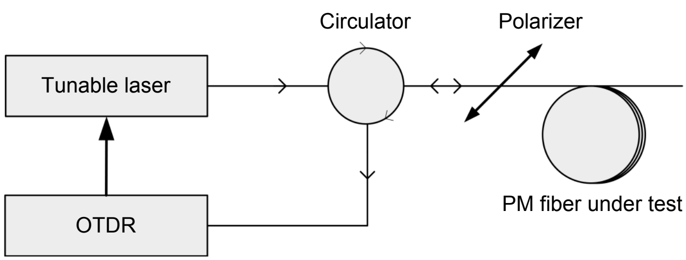

Fig. 2. Principle of measurement.

matrix $R = [-1,0;\ 0,-1]^T$, concatenation of the preceding sections (1 to $n-1$) is modeled as

$$T_{n-1} = \prod_{j=1}^{n-1} \alpha_j M_j R_j \tag{2}$$

where α_j is the loss of j-th element. Supposed the birefringence is homogeneous and polarization crossover is neglected, then the light intensity arriving at OTDR is

$$I(z) = \frac{5}{8} + \frac{3}{8}\cos(\Delta\beta 2z) = \frac{5}{8} + \frac{3}{8}\cos\left(\frac{2\pi}{L_B}2z\right) \tag{3}$$

From Equation (3), it is clear that the period of light intensity is half of beatlength L_B, and [7] used the period of light intensity in OTDR to measure mean birefringence of SM fiber, but it is not possible here for measurement of highly birefringent PM fiber due to the fact that L_B in PM fiber is very small (only a few millimeters), while OTDR's resolution is generally on the order of centimeter, so it cannot distinguish periodical variation of reflected light intensity and it is only a sampling of reflected light intensity. However, from a different expression form of $I(z)$ described by the following equation, we can find that in a frequency domain I is modulated by wavelength for a given location z:

$$I(\lambda) = \frac{5}{8} + \frac{3}{8}\cos\left(\frac{2\pi\Delta n}{\lambda}2z\right) \tag{4}$$

Therefore, for an element located at z, I varies a period with wavelength changing $\Delta\lambda$, then birefringence of this element is $\Delta n = \lambda^2/(2z\Delta\lambda)$. Wavelength can be varied with a high resolution, so distributed birefringence Δn may be acquired with a high precision.

3. Simulation results

Supposed each section in the concatenated model is 10 cm long, Δn or birefringence of each section is illustrated as Fig. 3**a**, the angle between each section which describes polarization crossover is a random value. Besides, a tunable laser varies from 1500 to 1560 nm with the step being 0.02 nm, and the resolution of OTDR is 10 cm. As a result, according to Eq. (1) the light intensity recorded by OTDR is calculated as Fig. 3**b**, where random noise has been considered to simulate situation in reality, the loss is neglected due to the small length of fiber, and birefringence's (Δn) dependence on wavelength is also neglected due to the fact that birefringence arises from stress difference. To resolve the distributed birefringence from the raw data in Fig. 3**b**, a correlation method is adopted to obtain the rough result of Δn without any

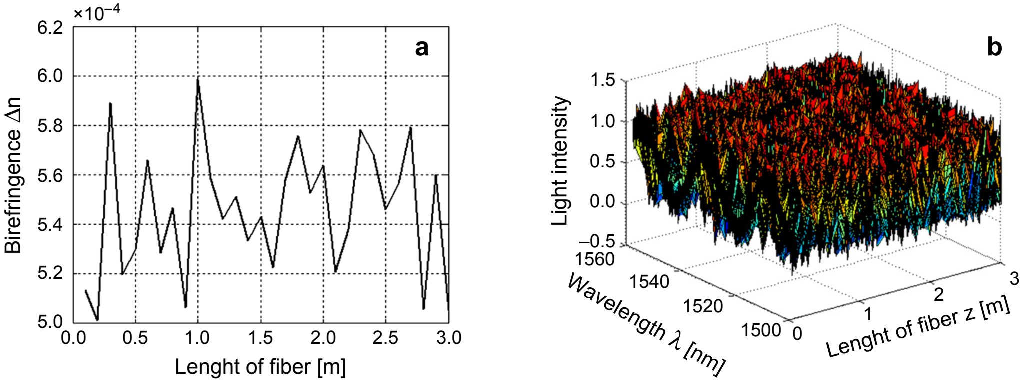

Fig. 3. Assumed birefringence and reflected light intensity at OTDR with noise considered. Assumed birefringence along a fiber (**a**), and calculated light intensity at OTDR (**b**).

consideration of polarization crossover. Then, make it as an initial value, and the least square method is adopted to obtain the accurate result of Δn with polarization crossover considered.

3.1. Analysis without polarization crossover

Considering non-homogeneous birefringence Δn and neglecting polarization crossover, the light intensity arriving at OTDR reflected from n-th element is

$$I_n(\lambda) = \frac{5}{8} + \frac{3}{8}\cos\left(\frac{4\pi}{\lambda}\sum_{j=1}^{n}\Delta n_j L_j\right) + I_N \tag{5}$$

where I_N is the noise to simulate a real situation. Therefore, the correlation method [9], a usual method to extract a weak signal from strong noise, is adopted for data

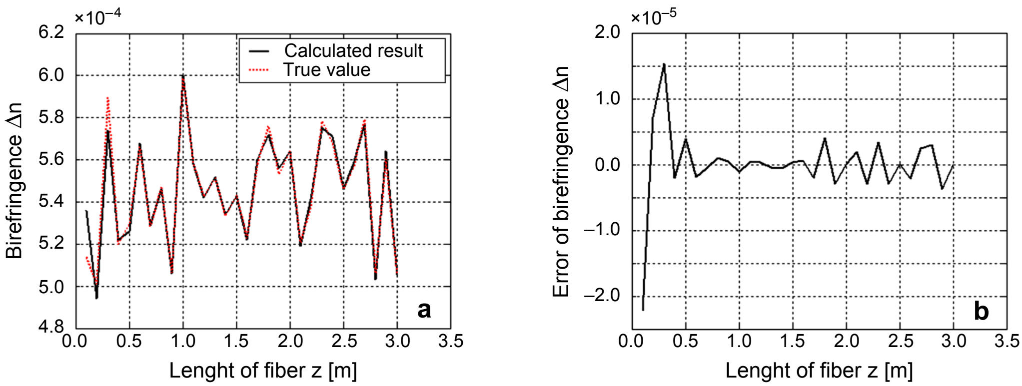

Fig. 4. Comparison between the true value and the calculated result of birefringence without polarization crossover considered. Calculated birefringence (**a**), and error between the calculated result and the true value (**b**).

processing. Supposed the birefringence Δn of sections (1 to $n-1$) has been already figured out, then the period of $I_n(\lambda)$ is only related to Δn_n. Reference signal I_R for the correlation method is chosen as:

$$I_R(\lambda) = \cos\left[\frac{4\pi}{\lambda}\left(\sum_{j=1}^{n-1}\Delta n_j L_j + \Delta n_n L_n\right)\right] \tag{6}$$

The correlation coefficient between $I_n(\lambda)$ and $I_R(\lambda)$ describes the degree at which Δn_n approaches the true value. Therefore, we change Δn_n to make the correlation coefficient get the maximum value which means Δn_n approaches the true value most. Finally, the distributed birefringence Δn has been calculated as shown in Fig. 4**a** with this method based on reflected light intensity recorded at OTDR. However, it is not accurate enough due to the influence of random polarization crossover, and the error is illustrated in Fig. 4**b**, which is about one order less than Δn. In fact, this result is enough for many occasions in reality.

3.2. Analysis with polarization crossover

When polarization crossover is considered, there exists a complex representation in the Jones vector and matrix, and it is complicated to acquire their parameters. Therefore, the Stokes vector and the Muller matrix are applied to calculate the complex relationship between reflected light intensity and birefringence of each element. The reflected optical wave from n-th element is [3, 10, 11]

$$E_{\text{out}} = P(M_{n-1}\ldots M_2 M_1)^T(M_n^T R M_n)(M_{n-1}\ldots M_2 M_1)E_{\text{in}} \tag{7}$$

where reflection's Muller matrix R, the polarizer P, and the Stokes vector of the optical wave E_{in} are given by the following equations:

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad E_{\text{in}} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

Supposed the Muller matrix of i-th element is M_i, $i = 1\ldots n$, [10, 11], and the product of M_1 to M_{n-1} is

$$M_{n-1}\ldots M_2 M_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & d & e & f \\ 0 & g & h & k \end{bmatrix} \tag{9}$$

Then, the reflected light intensity at the wavelength λ from n-th element is given by

$$I'_n(\lambda) = \frac{1}{2}\left\{A\cos^2(2\theta_n) + B\left[2d\sin^2(\delta_n)\cos(2\theta_n) + g\sin(2\delta_n)\right]\sin(2\theta_n) + - C\cos(2\theta_n) + D\right\} \tag{10}$$

where the birefringence of n-th section $\delta_n = 2\pi(\Delta n_n/\lambda)L_n$, $A = 2(a^2 - d^2)\sin^2(\delta_n)$, $B = 2a$, $C = 2dg\sin(2\delta_n)$, and $D = 1 - (g^2 - a^2)\cos(2\delta_n) + d^2$.

When the birefringence and the polarization crossover of elements (1 to $n - 1$) is known, there are only two undetermined parameters θ_n and δ_n (or Δn_n) in Eq. (10), which can be resolved with the combination of two equations $I'_n(\lambda_1)$ and $I'_n(\lambda_2)$. However, maybe the noise is so strong as to produce large error, so the least square method [12] is adopted here to utilize all of light intensity $I'_n(\lambda_i)$ to improve the accuracy of δ_n, where $i = 1, 2...m$, and m is the number of wavelengths in raw data of Fig. 3**b**. The square root of the residual error's quadratic sum is given by the following equation:

$$e = \sqrt{\sum_{i=1}^{m}\left[I'_n(\lambda_i) - I''_n(\lambda_i)\right]^2} \tag{11}$$

where $I''_n(\lambda_i)$ is the light intensity at the wavelength λ_i reflected from n-th section in Fig. 3**b**. The value of e describes the accuracy of θ_n and δ_n (or Δn_n). Therefore, based on the initial value of Δn_n obtained in Section 3.1, we change θ_n and δ_n (or Δn_n) to make e get the least value which means θ_n and δ_n (or Δn_n) approaches the true value most. Then much more accurate Δn can be resolved as illustrated in Fig. 5**a** due to the consideration of polarization crossover. The error (shown as Fig. 5**b**) is about two orders less than the true value of Δn.

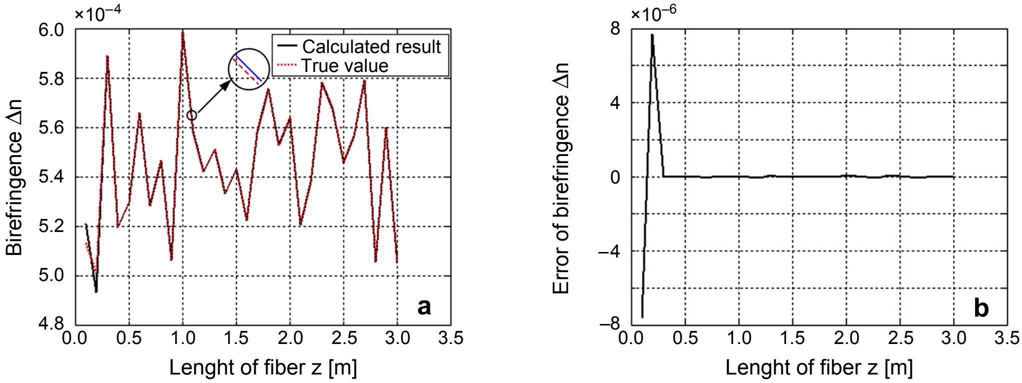

Fig. 5. Comparison between the true value and the calculated result of birefringence with polarization crossover considered. Calculated birefringence (**a**), and error between the calculated result and the true value (**b**).

4. Discussion

Reflected light intensity $I(\lambda)$ from an element located at z in PM fiber becomes a periodic signal when wavelength varies. $I(\lambda)$ varies a period with wavelength changing $\Delta\lambda$, then the birefringence of this element is $\lambda^2/(2z\Delta\lambda)$. Therefore, the resolution of distributed measurement depends on the resolution of OTDR, that is the distinguishable minimum length of fiber.

On the other hand, a variation step of wavelength is impossible to be infinitesimal. Therefore, the length of fiber L is limited as Eq. (12) for a given variation step of wavelength according to Shannon's theorem

$$L < \frac{\lambda^2}{4\Delta n \Delta\lambda} \tag{12}$$

What is more, polarization crossover is not our purpose but a factor affecting the precision of the calculated birefringence. However, it has also been resolved as an intermediate value in the method. Therefore, the method can also be applied to determinate approximately the distributed polarization crossover along a PM fiber.

5. Conclusions

A method allowing the determination of distributed birefringence in a PM fiber based on the modification of a traditional P-OTDR technique has been described. The resolution depends on that of OTDR, and the error is about one order less than the true value without consideration of polarization crossover, and about two orders less than the true value with polarization crossover considered. Simulations provide promising results, and we will establish a measurement setup in order to realize the method in the future.

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References

[1] WEGMULLER M., LEGRE M., GISIN N., *Distributed beatlength measurement in single-mode fibers with optical frequency-domain reflectometry*, Journal of Lightwave Technology **20**(5), 2002, pp. 828–835.

[2] HUTTNER B., REECHT J., GISIN N., PASSY R., VON DER WEID J.P., *Local birefrinence measurements in single-mode fibers with coherent optical frequency-domain reflectometry*, IEEE Photonics Technology Letters **10**(10), 1998, pp. 1458–1460.

[3] WUILPART M., MEGRET P., BLONDEL M., ROGERS A.J., DEFOSSE Y. , *Measurement of the spatial distribution of birefringence in optical fibers*, IEEE Photonics Technology Letters **13**(8), 2001, pp. 836–838.

[4] WUILPART M., *Method for Characterizing an Optical Fibre Link*, European Patent, 1811694 A1, 2007.

[5] YONGKANG DONG, LIANG CHEN, XIAOYI BAO, *Truly distributed birefringence measurement of polarization-maintaining fibers based on transient Brillouin grating*, Optics Letters **35**(2), 2010, pp. 193–195.

[6] MI JIAN, ZHANG CHUN-XI, LI ZHENG, WU ZHAN-JUN, *Measuring the beatlength of polarization maintaining fiber by broadband light source and conoscopic interference*, Journal of Optoelectronics-Laser **17**, 2006, pp. 1074–1077.
[7] CHEN XIN, MOZDY E.J., STAINER D.A., HUNT T.L., HERON N.A., *Method of Evaluating Fiber PMD Using Polarization Optical Time Domain Reflectometry*, European Patent, WO 2004/045113 A2.
[8] WUILPART M., ROGERS A.J., MEGRET P., BLONDEL M., *Fully-distributed polarization properties of an optical fiber using the backscattering technique*, Proceedings of SPIE **4087**, 2000, pp. 396–404.
[9] GAO JINZHAN, *Detection of Weak Signal*, Tsinghua Press, Beijing, 2002.
[10] PERLICKI K., *Calculation of birefringence distribution in optical fiber based on analysis of angle between states of polarization*, Optical and Quantum Electronics **37**(12), 2005, pp. 1141–1148.
[11] PERLICKI K., *Evaluation of the spatial distribution of birefringence in an optical-fiber link*, Microwave and Optical Technology Letters **42**(2), 2004, pp. 147–149.
[12] FEI YETAI, *Error Theory and Data Processing*, 6th Edition, China Machine Press, Beijing, 2010.

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