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MULTICRITERION AHP DECISION MAKING MODEL AS A TOOL FOR SUPPORTING THE SELECTION OF OPTIMAL DECISION IN A WATER SUPPLY SYSTEM

An AHP multicriterion hierarchy decision making process, based on linear aggregation of various criteria into a single multiattribute function is presented in the paper. The method is used if a number of factors, i.e., criteria and decision options, are to be assessed. By comparing pairs of criteria, a weight vector is determined which serves as a basis for the final selection. By comparing individual options the partial assessment matrices are constructed for each criterion. The method can also be used, for example, in environmental impact assessments, facility location selections, as well as planning and solving other environmental protection problems.

1. AHP METHOD

Decisions related to the technical design are of utmost importance because of their impact on costs, reliability and safety of operation; they also generate a damage risk. The decision making processes, especially those related to solving real complex problems, should be supported by formal analytic methods, e.g., optimizing ones. In technical problems, a number of opposite selection criteria should often be simultaneously taken into account. Among optimization methods the Analytical Hierarchy Process (AHP) plays a specific role. It is used for comparing decision options when a number of assessment criteria should be taken into account.

The other stages of the method are [1], [2], [3]: (1) setting a problem hierarchy, i.e., dividing the problem as a whole into problem groups of different importance levels and character, (2) assessing the importance of criteria (made by a decision maker), (3) evaluating criteria based options (made by an expert), (4) establishing aggregation rule and final assessment.

Stages 2 and 3 enable partial solutions to be obtained for individual hierarchy levels. They form pair-comparison matrices $\mathbf{A} = (a_{ij})_{ij}$ based on the so-called fundamental

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Saaty's nine-rank scale and set partial assessments (ω_j) at consecutive levels. Further, an object is considered as a criterion or option. According to Saaty's preference scale a_{ij} is assumed: 1 – if objects "i" and "j" are given the same preference, 3 – if object "i" is given higher preference than "j", 5 – if object "i" is given a considerably higher preference than "j", 7 – if object 'i" is given much more higher preference than "j", and 9 – if object "i" is given an absolute preference to "j". It is not recommended to use even intermediate values. The object's preference rank is to be assessed by the evaluating person and it results from the criterion importance or an extent to which the given criterion is fulfilled by the next option. In matrix **A**, where entries a_{ij} are results of appropriate comparisons, the following rules should be obeyed: (1) transitivity, i.e., if $a_{ij} > 1$ and $a_{jk} > 1$, so $a_{ik} > 1$, (2) correlation, i.e., beyond the diagonal $a_{ji} = 1/a_{ij}$ and (3) equality, i.e., on the diagonal $a_{ii} = 1$. It is also assumed that

$$a_{ij} = \omega_i / \omega_j, \tag{1}$$

where ω_i, ω_j – relative weights of assessment (for criteria or options). For the preference matrix developed in such a way, the following is true [2]

$$\mathbf{A\omega} = \boldsymbol{\lambda} \cdot \boldsymbol{\omega} \tag{2}$$

where $\mathbf{A} = (a_{ij})_{ij}$ – pair comparison matrix, $\boldsymbol{\omega}$ – eigenvector for matrix of weights ($\boldsymbol{\omega}_i$), $\boldsymbol{\lambda}$ – matrix eigenvalue. For matrix of such type all the eigenvalues except for the dominant value are equal to zero and $\boldsymbol{\lambda}_{\max} = n,$ where n is the matrix size. As measures of consistency of matrices, constructed by a decision maker and an expert, the following quantities are used [2]:

- Consistency Index

$$\mathrm{CI} = \frac{\boldsymbol{\lambda}_{\max} - n}{n - 1}. \tag{3}$$

- Consistency Ratio

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}}, \tag{4}$$

where RI is a random index value constant for the specified n [2]. The following preference stability conditions should be met:

$$\mathrm{CI} < 0.1 \quad \text{and} \quad \mathrm{CR} < 0.1. \tag{5}$$

Should preference stability fail, the pair comparison matrix ought to be verified. Eigenvectors shall be determined by the exponential method, while the maximum eigenvalue from the so-called Rayleigh's ratio.

Stage 4 requires a global aggregation rule *H* to be established to determine an option hierarchy for achieving the goal. A linear formula in the following form is commonly used [1]

$$H(Wi) = \frac{\sum \omega_j \cdot M_{ij}}{\sum \omega_j}, \tag{6}$$

where *Wi* – the *i*-th criterion, ω_j – weight of the *j*-th criterion, M_{ij} – partial assessment of the *i*-th option according to the *j*-th criterion, determined as an appropriate weight in step 3. The higher the value of the global criterion *H*, the better the specified option meets the requirements.

2. APPLICATION EXAMPLE

In a water supply system that consists of a single water intake, the conditions of water supply to the customers are not fulfilled completely. The system frequently fails and consequently provides insufficient amount of water or water of improper quality to the customers. The most important problems of the water supply company (WSC) are: frequent undesired events related to a single water source, i.e., frequent surface water contamination caused mainly by manufacturing plants located upstream the water intake, problems with treating water up to required quality standards in the case of such events, high failure frequency of the distribution network, considerable water losses in the network, secondary water contamination caused primarily by a lower water flow rate in pipes due to reduced water use. The raw water monitoring and the raw and treated water reservoirs are examples of security measures. If a random non-removable water contamination is detected, the intake is closed and put out of service. In the system under consideration the protecting measures used are insufficient.

Stage I is a hierarchical process. At this stage, the following are distinguished: (1) establishing the goal of decision making support, (2) establishing assessment criteria, and (3) establishing alternative solutions (options).

The aim of the WSC management is to improve water supply to customers. The following were assumed as the criteria of the best option: K1 – improved water quality at customers, K2 – higher reliability of the water supply system, indicated directly by, e.g., shorter break in water supply, K3 – the lowest cost necessary to achieve the required effect. To improve the existing situation the following options are considered: W1 – replacing some pipes, chiefly main ones, characterized by considerable failure risk, considerable risk of secondary contamination or significant water losses, W2 – use of another, additional water source that is unexposed to contaminations, W3 – modernization of the existing water treatment plant (ZUW).

The option and criterion sequence is not related to decision maker’s preferences.

The option can be completed on a whole only.

Stage II is an option comparison made by the decision maker. In this case, the improvement of water quality supplied to customers K1 is of utmost importance. The partial assessments made by the decision maker specify in a sufficient way the elements of matrix **A**: $a_{12} = 3$, $a_{13} = 5$ and $a_{23} = 3$. For the matrix thus specified the eigenvector after normalization is $\omega = (0.637; 0.258; 0.105)$. The elements of this vector indicate the weights of individual criteria in the final linear assessment. As one can see, the highest weight is assigned to criterion K1 ($\omega_1 = 0.637$). The criterion K3 is of the lowest significance. The dominant eigenvalue corresponding to the determined eigenvector is $\lambda_{\max} = 3.0385$. To verify the correctness of preferences, the control values were computed: consistency index CI = 0.019 and consistency ratio CR = 0.033. Since conditions (5) are met, the stability of preferences remains untouched.

Table 1

Expert's comparison table

	K1			K2			K3		
	W1	W2	W3	W1	W2	W3	W1	W2	W3
W1	1	1/5	1/6	1	1/5	1/3	1	1	1/5
W2	5	1	2	5	1	3	1	1	1/5
W3	6	1/2	1	3	1/3	1	5	5	1

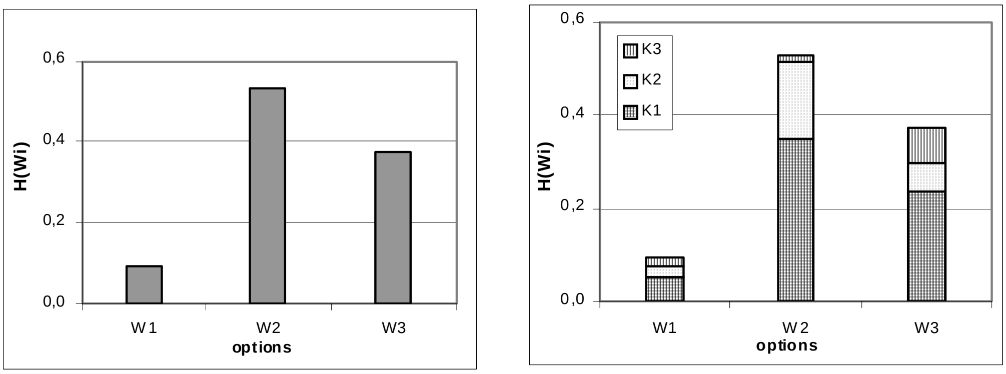
Stage III is an option comparison based on the adopted criteria made by an expert. The result of the option comparison process with respect to each of the three criteria are matrices **A**(*Ki*), $i = 1, ..., 3$, as presented in table 1. For each matrix **A**(*Ki*) containing the results of option pair comparison with regard to each criterion, the normalized eigenvectors and corresponding dominant eigenvalues are listed in table 2. In all the cases, the consistency conditions have been fulfilled (CI < 0.1 and CR < 0.1). The other eigenvector components $\lambda_{\max}$ reflect the relative weights of appropriate options with respect to particular criteria. As one can see, for criteria K1 and K2 the best option is W2, while W3 is the best option for criterion K3.

Table 2

Partial solution comparison of options vs. criteria

	Matrix eigenvectors	Eigenvalue $\lambda_{\max}$	Consistency Index CI	Consistency Ratio CR
K1	(0,082; 0,550; 0,368)	3.086	0.0429	0.074
K2	(0,105; 0,637; 0,258)	3.038	0.0193	0.032
K3	(0,143; 0,143; 0,714)	3.000	0	0

Stage IV serves to derive the final assessment. For linear aggregation (6) the following option hierarchy was obtained: *H*(W1) = 0.094; *H*(W2) = 0.530; *H*(W3) = 0.376. The option W2, that was assigned the highest note, is the best one. It is possible to present graphically both a comparative analysis and individual share of particular partial assessments in the overall assessment (the figure). One can conclude that criterion K1 had the strongest impact on the final assessment.



Weighed final option assessment (without and with particular partial assessments)

Additionally, an analysis of solution sensibility to preference changes in comparison matrices was carried out. In the case of poor financial conditions, it is the costs that became the most important factor. A significant change of decision maker ($a_{12} = a_{31} = a_{32} = 3$) while expert's assessments remained unchanged led to solution change. Option W2 became the best one.

The example presented was simplified, as the aim of this paper was not to solve the existing problem but to present the AHP multicriterion method and to show its suitability for solving complex decision making problems that occur also in water supply systems. In order to use this method it is necessary to gain just estimated values, since the multi-rating assessment scale enables "fuzzy" consideration of appropriate preferences.

At fixed expert's assessment, the results depend on a subjective evaluation of criteria made by the decision maker. This stage is considered to be the most important one, as a "predecision" based on preferences embedded in the decision maker's mind.

The AHP method is suitable especially when some criteria of option assessment are not quantitative but qualitative and considerable part of assessments are subjective ones. This method is considered to be reliable due to considering the decision maker's preferences and low sensibility to assessment errors (even at a large number of comparisons any inconsistencies and errors cause that the consistency conditions (5) are not met).

3. SUMMARY

1. Intuitive selection in multicriterion problems is sometimes a difficult task. The mathematical methods supporting decision making formalize and systematize the process, while ensuring a logical consequence of the selection made. Analytical Hierarchy Process (AHP) also called Saaty's method is one of the multicriterion optimization methods.

2. The AHP method is based on actual assessments and the decision maker's preferences. It assumes comparability of all criteria, both measurable and immeasurable ones. It allows assessment subjectivity and uncertainty to be taken into account, when available data are of general character. By computing additional indices (CI, CR) it is possible to evaluate inconsistency of partial assessments.

3. The methods enables option arranging, based on the global criterion H introduced as an aggregated utility function. The linear aggregation formula (6) is the most often used one. The final assessment indicates *a posteriori* preferences.

4. The AHP method, originating from operative methods, has not only advantages (effective, transparent, easy to operate) but also faults (e.g., rapid increase in the number of necessary comparisons at a relatively low increase of criteria or options).

5. Currently, the AHP method is considered to be the most general-purpose tool for multicriterion assessment. It can be suitable for solving difficult problems in environmental engineering.

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WIELOKRYTERIALNY MODEL DECYZYJNY AHP JAKO NARZĘDZIE WSPOMAGAJĄCE WYBÓR OPTYMALNEJ DECYZJI W SYSTEMIE ZAOPATRZENIA W WODĘ

W artykule przedstawiono wielokryterialny hierarchiczny model decyzyjny AHP oparty na liniowej agregacji różnych kryteriów w jednej wieloatrybutowej funkcji. Metodę stosuje się, gdy ocenie należy poddać wiele czynników, tj. wiele kryteriów i wariantów decyzyjnych. Porównując parami kryteria, wyznacza się wektor wag stanowiący podstawę ostatecznego wyboru, a porównując warianty, tworzy się macierze ocen cząstkowych względem każdego z kryteriów. Metoda może być stosowana m.in. w ocenie oddziaływania inwestycji na środowisko, wyborze lokalizacji obiektów, planowaniu i rozwiązywaniu innych problemów ochrony środowiska.