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Theoretical study of image patterns

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1. Introduction

Recently, an extensive study [1] has been made on the statistical properties of speckle phenomena. Asakura [2–5] et al. studied the effect of the object surface roughness properties on the speckle pattern. The usefulness of the fundamental technique using speckle pattern for measurement of the surface roughness properties of the object has been reported by Fujii et al. [6, 7] conducted a computer study on the intensity distribution and contrast variation on image speckle patterns for object having different surface profiles.

In this work we study the intensity distribution for object having a parabolic shape as a function of the surface roughness properties of the object and the point spread of an imaging system.

2. Theoretical analysis

For mathematical simplicity only one dimensional case is treated in the present work. By neglecting the absorption loss due to the transmission (or reflection) of light through the object, the intensity distribution of an image is given by:

$$I(x') = \Big| \int_{-\infty}^{\infty} K(x'-x) \exp\{i\Phi(x)\} dx \Big|^2 \tag{1}$$

or, in symbolic notation, then

$$I(x') = |K(x') \otimes e^{i\Phi(x')}|^2$$

($\otimes$ stands for convolution operation), where $K(x)$ is the amplitude point spread function of the optical imaging system and $\Phi(x)$ stands for the phase variations of the object surface [2] from which the image appears.

The point spread function $K(x)$ is assumed to take a simple rectangular form expressed by:

$$K(x) = \begin{cases} 1, & |x| \leqslant a, \\ 0, & |x| > a. \end{cases} \tag{2}$$

The intensity distribution of the image given by Eq. (1) becomes

$$I(x') = \Big[\int_{x'-a}^{x'+a} \cos\{\Phi(x)\} dx \Big]^2 + \Big[\int_{x'-a}^{x'+a} \sin\{\Phi(x)\} dx \Big]^2 . \tag{3}$$

In the present work $\Phi(x)$ takes the form

$$\Phi(x) = kx^2$$

where k is determined by the surface roughness R as follows

$$\Phi(x = \delta) = R = k\delta^2, \text{ i.e., } k = R/\delta^2,$$

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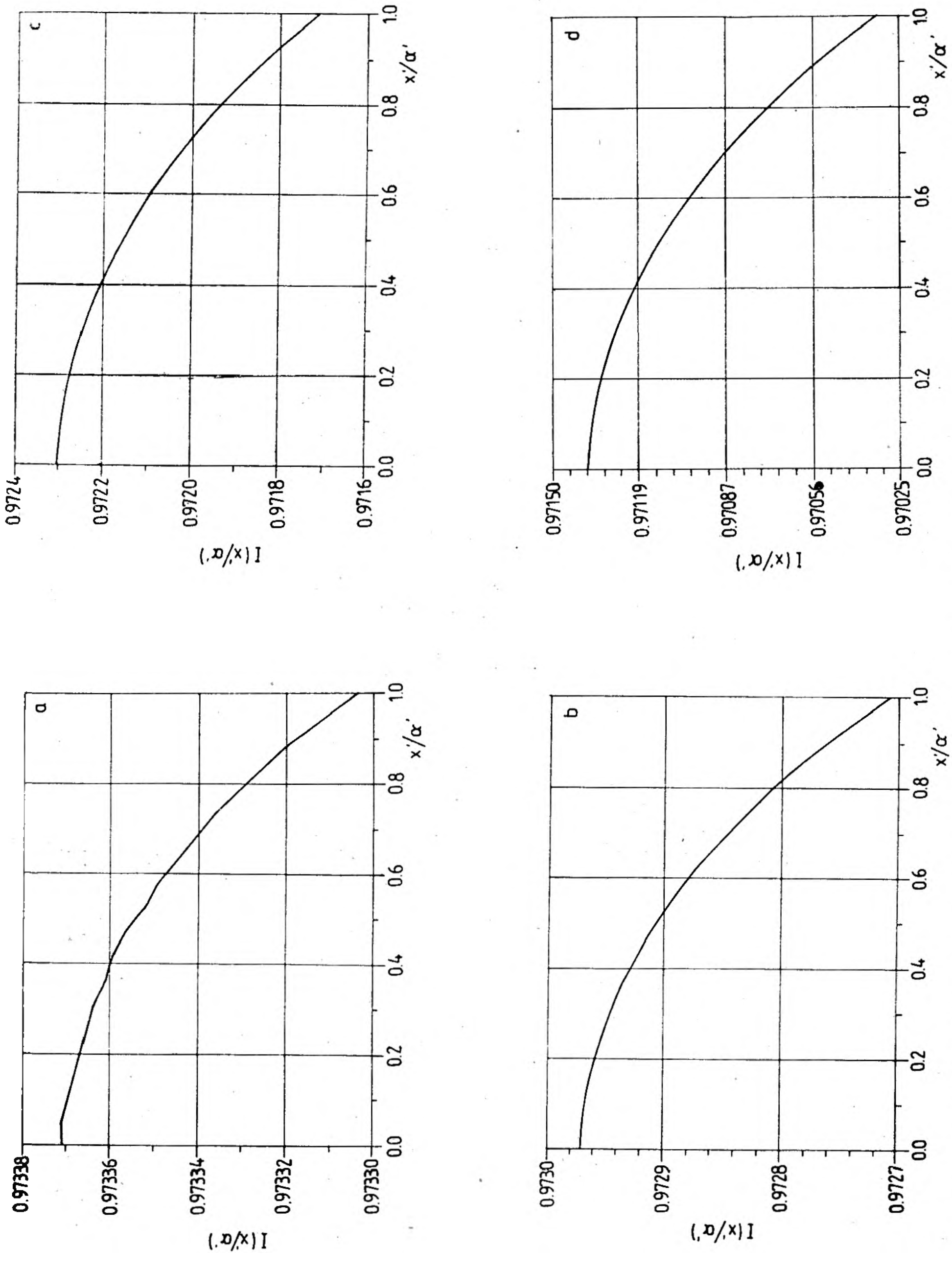

Fig. 1. The intensity distribution $I(x'/a)$ for parabolic object of surface roughness. R = 0.04 μm (a), 0.08 μm (b), 0.12 μm (c) and 0.16 μm (d); $a/a' = 1$

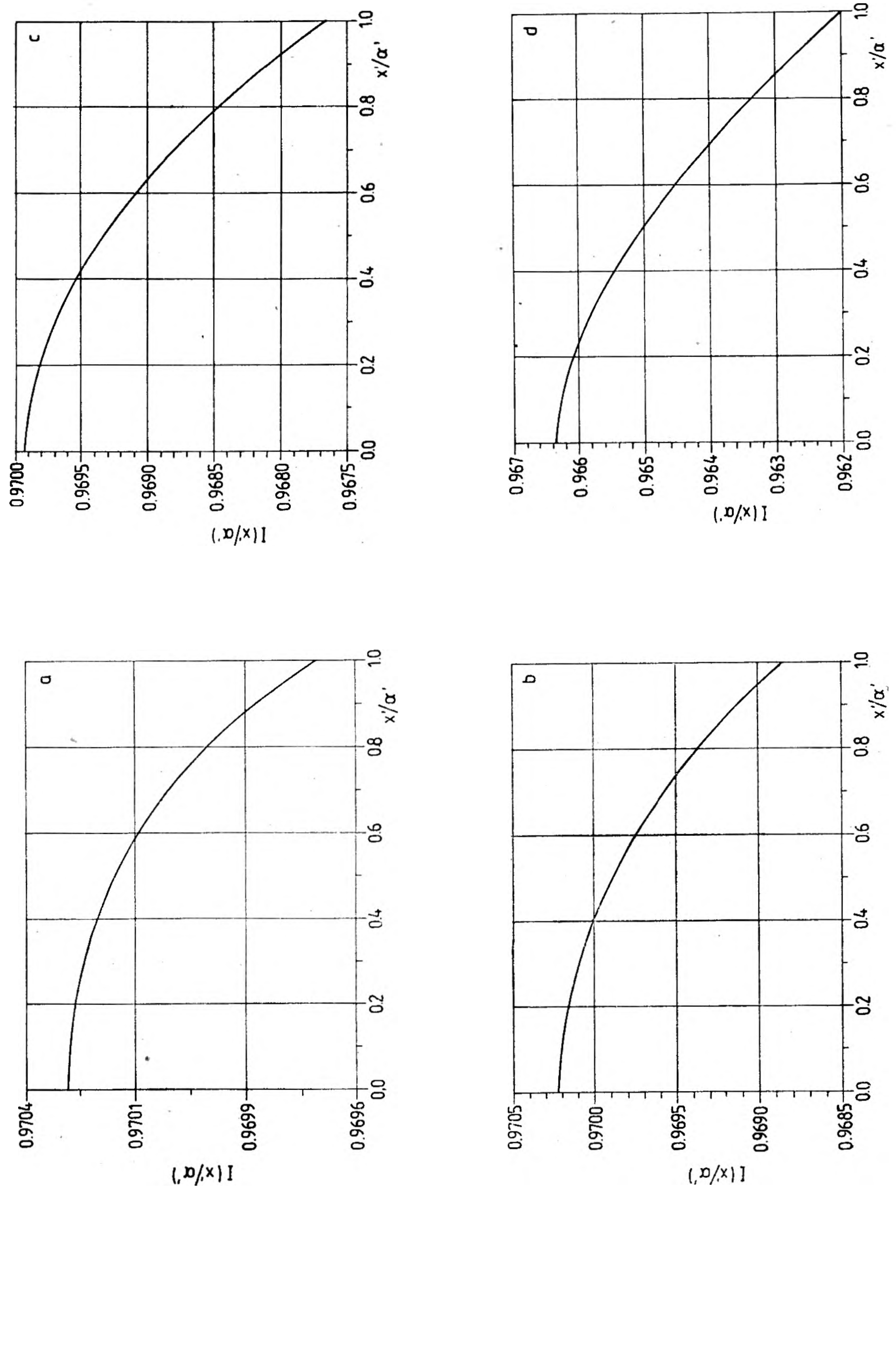

Fig. 2. The intensity distribution for object having parabolic shape under different imaging conditions of the point spread $a/\alpha' = 0.572$ (a), 0.92 (b), 1.226 (c) and 2.075 (d); $R = 0.2$ μm

taking $\delta = a'$, we have got

$$\Phi(x) = R \times \left(\frac{x}{a'}\right)^2, \tag{5}$$

hence

$$I(x) = \left[\int_{x'-a}^{x'+a} \cos R \times \left(\frac{x}{a'}\right)^2 dx\right]^2 + \left[\int_{x'-a}^{x'+a} \sin R \times \left(\frac{x}{a'}\right)^2 dx\right]^2. \tag{6}$$

In the case $q/a' = 1$

$$I(x'/a') = \left[\int_{x'-a'}^{x'+a'} \cos\left[R \times \left(\frac{x}{a'}\right)^2\right] dx\right]^2 + \left[\int_{x'-a'}^{x'+a'} \sin\left[R \times \left(\frac{x}{a'}\right)^2\right] dx\right]^2 \tag{7}$$

where a' is the correlation length of the object.

3. Results

The intensity distribution $I(x'/a')$ of image patterns for object having parabolic surface is calculated for different surface roughness of $R = 0.4$, 0.08, 0.12 and 0.16 μm as shown in Figs. 1a–d. In this case, the point spread a of the imaging system is set equal to the correlation length a' ($a/a' = 1$). Figures 2a–d show the intensity distribution of image patterns from an object having the surface roughness $R = 0.2$ μm produced under different imaging conditions of the point spread $a/a' = 0.572$, 0.92, 1.226 and 2.075.

4. Conclusion

From Figure 1 we can infer that the image intensity variation is enhanced with the increasing surface roughness of the object. Also Figure 2 clearly indicates that the image intensity variation is enhanced with the increasing point spread a of the imaging system.

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