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Biomechanical model of human eyeball and its applications

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Attempts at the mechanical identification of the human eyeball are often not very effective for two reasons: the material parameters determined by tension tests on corneal and scleral tissue specimens are not sufficiently accurate while numerical models of the eye, integrating material and geometric parameters, are often based on unrealistic assumptions. The examples presented here cover refractive surgery, Goldmann applanation tonometry and the optical self-adjustment of the eye. The discussed problems are illustrated with calculations showing that it is possible to effectively use a biomechanical model of the eye to identify its material parameters. Also the handicaps, the Imbert–Fick law among them (numerical calculations do not corroborate this law), lying at the basis of applanation tonometry are demonstrated. The conclusions can help to create a realistic numerical model of the eyeball.

Keywords: eyeball, biomechanical model, tonometry, intraocular pressure (IOP), optical self-adjustment.

1. Introduction

There has been a growing interest in the numerical modelling of the human eyeball in view of its practical applications in tonometry and refractive surgery. The mechanics of the outer shells of the eye is related to optical system since their most flexible part – the cornea – is at the same time the strongest lens in this system. The intraocular pressure – usually the principal load acting on this shell structure – is subject to variation in a relatively wide range, noticeably affecting the displacements of the cornea and the sclera. The deformations translate into changes in optical power and the consequent shifts of the optical focus relative to the fundus of the eye, affecting the sharpness of the image on the retina. Thus the eyeball's structural (geometrical and material) parameters have a major influence on its optical functions. In recent decades this influence has been exploited to correct the optical power of the eye by surgically altering the profile of the cornea's outer surface. Complete knowledge about the geometry of the eyeball and the material parameters of the tissues forming it would make it possible to precisely plan the effects of such surgical procedures.

Despite the fact that attempts at the mechanical identification of the eye have been made for nearly a century, only the physicochemical structures of the cornea and the sclera, and recently also their geometries, have been identified. The mechanical parameters of primary importance for solutions concerning displacements, *i.e.*, moduli of elasticity, are still the subject of controversy.

2. Structure of eyeball model

Ophthalmologists distinguish many layers in the cornea, but only the stroma plays a principle role in eyeball mechanics. The stroma takes up over 90% of the corneal thickness. For this reason the material of the cornea is equated with this single kind of tissue. Because of its specific structure, the shell is globally isotropic in the directions tangent to the middle layer. Moreover, because of its outer shape and material characteristic the cornea can be treated as a membrane. Therefore in calculations this tissue is usually treated as an isotropic material [1]. Similar simplifications are applied to the sclera.

In the corneal-scleral shell the transitional zone, called the limbus, plays a special role. The material of the limbus shows noticeable anisotropy [2], but the area which it occupies between the cornea and the sclera is so small that the anisotropy of this zone is usually neglected. But the ciliary body together with the muscle which controls the lens as well as the choroid cannot be neglected. These tissues clearly increase the overall limbus and sclera stiffness and so this effect should be taken into account in investigations of eyeball deformations caused by intraocular pressure.

3. Material constants

3.1. Exponential characteristic

Although the cornea material curve is still sometimes approximated by a linear function, today both cornea material nonlinearity and anisotropy (and even rheology) are commonly taken into account. However, not always the above effects and not all of them at the same time must be taken into account.

The elastic nonlinearity of the stroma and that of the sclera, under uniaxial stress, is usually described by the exponential relation proposed by Woo *et al.* [3]:

$$\sigma = A\left[\exp(\alpha\varepsilon) - 1\right]$$

where: σ – stress, ε – strain, A and α – material constants. For a complex state of stress one should additionally reduce the main stress tensor components to uniaxial stress [4] according to the formula:

$$\sigma^* = \left\{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2\right]\right\}^{0.5}$$

The main strain tensor components are converted in a similar way

$$\varepsilon^* = \left\{ \frac{2}{9} \left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2 \right] \right\}^{0.5}$$

3.2. Longitudinal modulus of elasticity

The material of the eye shells is often described by a secant elasticity modulus, *i.e.*, a ratio of total stress to total strain in a uniaxial state of stress. This parameter can be compared for different materials at a fixed level of stress. In the case of the cornea material, the (two-axial) steady-stress component in the apex, amounting to about 20 kPa at the nominal intraocular pressure, can be considered to be such a level. The elasticity modulus E (measured *in vitro* on samples excised from the cornea) found in the literature, ranges widely from 0.026 MPa [5] to 57 MPa [6] and to as much as 115 MPa reported by UCHIO *et al*. [7].

4. Biomechanical models of eyeball

The eyeball as a structure seems to be uncomplicated and relatively amenable to mechanical analysis. This is really so in some special applications, *e.g.* when modelling the effects of dynamic loads caused by quickly moving glass slivers or an air bag impact. In other applications, *e.g.* in refractive surgery, the most important aspect of the eye model is often its optics and then the analysis does not end with the determination of the displacement field and the stress field as in the case of, let us say, the thigh bone. The configuration of the loaded structure requires further calculations to determine the change in the position of the optical focus relative to the fundus caused by deformation. The results of such calculations are highly sensitive to displacement solution accuracy and so to the preliminary geometry and material assumptions, the simplifications made and the adopted boundary conditions. It is extremely difficult to obtain correct results and, in the author's opinion, they are rare exceptions in the literature on the subject.

The models found in the literature fall into two groups: analytical models [8, 9] and numerical models [1, 10–14, 17]. An analytical model would have this advantage over a numerical model that by providing a closed solution it would make it possible to investigate the influence of individual parameters on its optical functioning. Thus any effects of changes in the parameters (*e.g.*, an increase in intraocular pressure resulting in a change in optical power) would have a physical justification. Unfortunately, the current analytical capacities in this regard are insufficient and models which can be solved in this way are limited to the cornea (with a constant thickness and made of a linear material) alone, which is too large simplification. The numerical model has no such limitations. It can cover the whole eyeball (the cornea together with the sclera) with any geometry and it can be equipped with nearly any material: anisotropic, nonlinear or inelastic. Such boundary conditions (the way in which the model is fixed) do not encounter any computing barriers. Thus the model's potential seems to be limitless.

But the numerical model has one major drawback – the solution it provides is in the form of numerical tables. Each relation investigated in this model requires a series of separate solutions and the obtained functions come from approximations. The latter, however, are performed arbitrarily and so have no physical justification.

Despite the above inconveniences, the numerical model of the eyeball has become a powerful investigative tool. Thanks to computer tomography, the geometry of all the structural details of the eye has been precisely determined. But little is known about the mechanical properties of the tissues forming the eyeball. The longitudinal moduli of elasticity measured by different researchers vary by as much as four orders (see Sec. 3.2) and it is difficult to distinguish between correct and worthless results. A similar scatter characterizes the other measured mechanical parameters. We have found ourselves in a rather uncomfortable situation when the possibilities offered by numerical techniques have got much ahead of our laboratory potential as regards the investigation of the mechanical properties of the eye's tissues and its optical functions. The results yielded by the tensile test turn out to be so uncertain that researchers have turned to the numerical model of the cornea and the sclera to identify the tissues in the mechanical respect. The model material or geometry parameters are matched to make the model behave in the same way as the real eye. By imposing

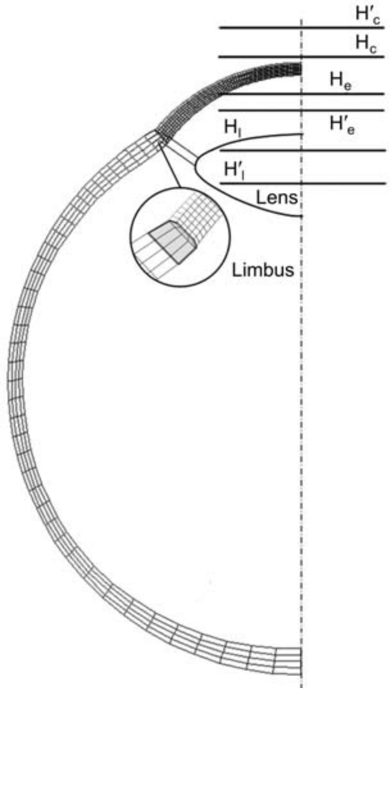

Fig. 1. Schematic finite element model of the eyeball with principal planes of the eye optical system.

constraints on the parameters, one can determine their proper ranges. This approach has turned out to be more effective. An axial symmetric finite element eyeball model, on which this kind of material identification was made, is described in [13]. The model is built from 2D solid quadrilateral 8-node body of revolution elements shown in Figs. 1 and 3.

5. Numerical identification of corneal material

The eyeball models found in the literature were designed mostly to simulate the flattening of the corneal apex in Goldmann applanation tonometry [8, 10, 11] or the change in the eyeball's optical power after surgical correction of its geometry [1, 12, 14]. In both cases, the model is also used to identify the cornea material. The results are often far from ideal because of the questionable assumptions or outright errors made in the creation of the model. Despite this, the obtained parameter values are within a much narrower range than the ones determined experimentally. Sometimes they are quite reliable, although obtained on the basis of flawed assumptions or after gross simplifications. Numerical models seem to be much more researcher-friendly than biological preparations. It often happens that a model which was not previously verified is used to identify the cornea material or to investigate the influence of its thickness on tonometrically measured intraocular pressure. The trust placed on the model by its creator sometimes seems to be boundless. The few problems of model's applications in ophthalmology are described below.

5.1. Refractive surgery

The model is verified by introducing the same changes as the ones made by surgery into the geometry of the cornea and then calculating its radius of curvature in the apex under intraocular pressure. The change in optical power calculated for the model is compared with the change observed after the surgery. If the model is correct, the respective results, evaluated by the quality of the image on the retina, should be similar.

Geometry modifications made by photorefractive keratectomy (PRK) are the easiest to introduce into the model. The surgery performed with a laser consists in changing the curvature radius of the corneal apex (over a diameter of about 7 mm) through ablation (vaporization) of its outer layers. During the operation the intraocular pressure does not change. If no astigmatism is corrected, but only optical power, the cornea after the new profile is introduced, still remains (approximately) axially symmetric. It seems quite easy to create a model and numerically solve the problem. Unfortunately, "it seems" is the most certain element in this thesis.

An attempt at a numerical solution encounters a formidable difficulty for quite an inconspicuous reason. An eyeball model always comes into existence through a design, *i.e.*, a configuration of the structure prior to loading. This applies to the geometry before and after the operation. The problem derives from the fact that in clinical conditions the geometry in both cases is unavailable. Only the final configuration is known, *i.e.*, the dimensions of the cornea under pressure before surgery and the postoperative dimensions of the cornea deformed by both the surgery and the intraocular pressure. Only after the solution one can find out whether the cornea model, both the one before the surgery and the one after the surgery (they are two different models), acquired correct dimensions, but the configuration of the model under load depends on both the initial geometry and the assumed material elasticity. The injustice which the analyst suffers here consists in the fact that the surgeon is completely unaware of the problem – the measurements before the surgery, the surgery itself and the postoperative checkup are conducted at a (roughly) invariable intraocular pressure. The geometry of the cornea not subjected to load does not occur here at all.

One can easily guess what the basic error in the numerical PRK simulation is – the change in the eye's optical power is calculated for a "frozen" eyeball. The model (not subjected to load) acquires eyeball dimensions from clinical measurements, *i.e.*, carried out on the eyeball subjected to load. Then it is being solved (changes its dimensions, and so also its optical power) and the geometry correction caused by PRK is introduced into the obtained model which is now considered to be stiff. For this final model configuration the ultimate optical power of the cornea is calculated. Is such an algorithm admissible?

The answer depends on the cornea's elasticity modulus. If it is close to 8 MPa, as indicated by many reliable experiments, carried out mainly by Hjortdal [2], the answer is yes since the shell characterized by this elasticity modulus is so stiff that a change in pressure from 0 to 2.135 kPa (16 mmHg) has little influence on its configuration. If, however, the elasticity modulus is close to 0.3 MPa, as indicated by other equally reliable experiments [7, 8], including ours [13], then the answer is no. One could get the impression that this question is decided by a vote if it were not for the fact that the former figure comes from measurements while the latter in most cases is the result of numerical simulations. The existing experience suggests that the latter figure is correct. The role which the cornea's elasticity modulus plays in predicting changes in the optical power of the model after PRK and the significance of the simplifications made in such calculations were investigated by the authors in [14].

Radial keratotomy is an example of another difficulty, this time associated with the cornea material itself. The surgery is performed using a special scalpel and consists in making several deep incisions arranged radially on the peripheries of the cornea, as is seen in Fig. 2. The apex flattening caused by intraocular pressure corrects myopia. To the above difficulties, the numerical simulation of the surgical procedure adds another one: this kind of change in the geometry of the cornea results in high stress gradients on the bottom of the incisions. Then the assumption that the material is anisotropic seems untenable. But one can easily find attempts at such solutions [1, 12] and it is by no means certain that they are basically flawed.

5.2. Goldmann applanation tonometry

The numerical simulation of intraocular pressure (IOP) measurement by means of an applanation tonometer is readily used to verify the cornea material adopted

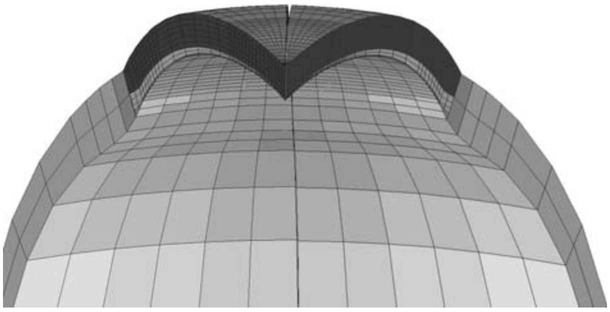

Fig. 2. Part of the cornea after modelling of the radial keratotomy.

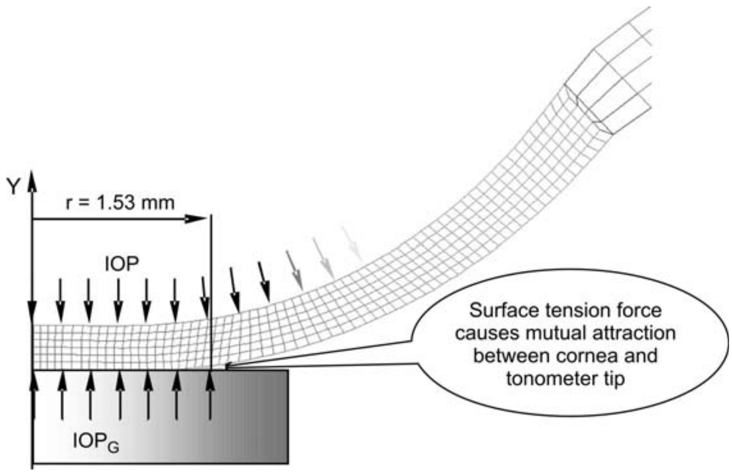

Fig. 3. Cross-section of cornea. *Y*-axis is symmetry axis. Surface tension force reduces resultant IOP_G force needed to flatten corneal apex over radius r = 1.53 mm. Forces originating from IOP and from reaction of part of cornea surrounding disc act from inside on flattened disc.

in the model. This measuring method [15] is based on the assumption of equality of pressures on both sides of the flattened corneal apex, *i.e.*, the external pressure (denoted as IOP_G) caused by the (flat) measuring tip of the instrument and the internal pressure (denoted as IOP):

$$IOP_G = IOP \tag{1}$$

The former pressure is associated with the name of Goldmann. After the pressure force and the diameter of the zone of contact between the tonometer tip and the cornea are measured, the average IOP_G pressure is calculated. Eq. (1), called the Imbert–Fick law, holds good, according to Goldmann, only for the so-called calibration dimensions:

- applanation zone diameter D = 3.06 mm,
- apex cornea thickness CCT = 0.52 mm,
- apex cornea curvature radius R = 7.80 mm.

The measurement of pressure on a cornea with dimensions other than the calibration ones requires a correction for CCT and R. But the dependence of IOP_G on the pressure level and the cornea material parameters is not taken into account.

Applanation tonometry has an over one hundred years old tradition and it would seem that the theoretical basis of the mechanics of this measurement is well understood. However, some arguments and beliefs expressed in the literature, although they do not arouse controversy there, are at odds with our numerical solutions. An example here is the argument for calibration dimensions put forward by Goldmann and Schmidt [15] and repeated by others [16]: justifying diameter $D = 3.06$ mm it is assumed (Goldmann did not do any such calculations) that externally applied pressure IOP_G acting over diameter D is counteracted from the inside by pressure IOP and by the bending resistance of the shell not exposed to load. Thus IOP_G is always higher than IOP and condition (1) is satisfied only when also the force of attraction between the tonometer tip and the cornea is taken into account. The force originates from the surface tension in the lacrimal film connecting the two surfaces and its magnitude counterbalances the forces bending the shell exactly at $D = 3.06$ mm. As is apparent, the result is understood here as a superposition of two solutions: 1) for a shell (membrane) devoid of bending rigidity – then the external pressure and the internal pressure, acting in the applanation zone, are exactly equal, regardless of its dimensions and the value of IOP and 2) for a shell with low bending rigidity, loaded by only this (constant) part of pressure IOP_G which is needed to bend the shell at IOP = 0.

Our studies of the eyeball model indicate that the above assumption is incorrect and leads to not only large errors, but also to a paradox. The problem is best illustrated by the solution obtained for a linearly elastic model. But the most serious consequences follow from the solution for a realistic nonlinear model. The two results are presented as graphs in Fig. 4. Each of the models (the linear model and the nonlinear one) has such a cornea material that at nominal intraocular pressure IOP = 16 mmHg the calculated average pressure exerted from the outside by the tonometer tip, IOP_G, also amounts to 16 mmHg. According to current applanation tonometry, the measured pressure IOP_G is a linear function of IOP, represented by grey dashed line in the figure, *i.e.*, written as Eq. (1). If the force originating from surface tension were neglected (as it is done in numerical models), then the function graph would be a grey dashed line vertically shifted by the initial value of IOP_G (at IOP = 0).

The solution for the linear model, shown in Fig. 4, is, of course, far from the reality, but it clearly puts in doubt the applanation tonometry's assumption that the difference between IOP_G and IOP does not depend on IOP. As the diagram for the linear model shows, the influence of the intraocular pressure is so strong that even the functional trend between the variables has been reversed. One can say that in the linear model by increasing IOP one helps IOP_G to flatten the apex.

According to Fig. 4, the nonlinear model behaves quite differently in this respect. But the functional dependence for this model does not coincide with the applanation tonometry predictions. At a low IOP (below the nominal value) applanation pressure IOP_G is, as assumed by Goldmann, actually higher than IOP. But as the pressure

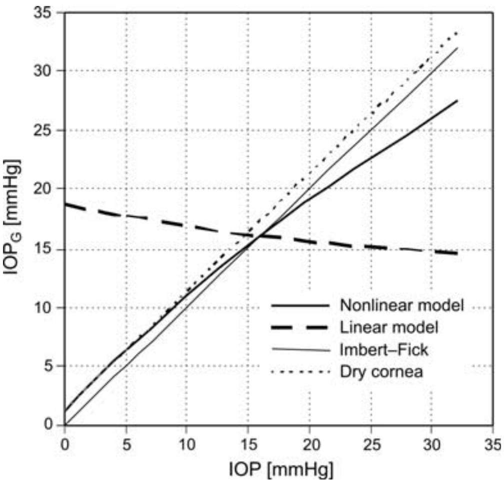

Fig. 4. Calculated IOP_G for model made of: linear-elastic material and nonlinear-elastic material, depending on IOP. Grey lines represent Goldmann idealization: broken line – pressure measured on dry (not wetted with lacrimal fluid) cornea, solid line expresses Imbert−Fick law (Eq. (1)) and so takes into account the influence of surface tension in lacrimal film. All bold lines satisfy condition $IOP_G = IOP = 16$ mmHg (nominal).

increases, a trend similar to the one observed for the linear model emerges: the higher the IOP, the easier it is to flatten the corneal apex. Thus above the nominal pressure, IOP_G becomes lower than IOP. This is the paradox: the pressure on the outside of the flattened disc is lower than the pressure acting from the inside. If Goldmann's assumption about the influence of the shell's bending resistance and the surface tension were correct, the cornea's bending resistance in this case would have to be negative! The rest of the cornea then would attract the flattened disc, instead of repelling it.

This result stands in contradiction to all the theses and the experimental results found in the literature on the subject. All the opinions and measurement data support the above applanation tonometry assumptions and so they are at variance with the result presented here. This raises questions about the quality of the model and the sense of the obtained solutions. Can the bending resistance of a spherical shell being flattened and simultaneously loaded with an external pressure be negative? However strange the answer will sound – this is precisely the case.

The applanation tonometry's cornea deformation model based on intuition has never been verified before. To the author's knowledge, the calculation results shown in Fig. 4 are the only attempt to check Goldmann's assumptions. The numerical model, described in [13, 14], used for this purpose, was carefully prepared with regard to both the selection of materials for the cornea, the sclera and the corneal limbus and its agreement with the known experimental results. According to Fig. 4, applanation pressure IOP_G for the model not loaded with internal pressure is slight, amounting to about 1 mmHg. Thus there are no reasons to question the IOP_G value obtained at IOP = 32 mmHg. The calculated $IOP_G = 27.5$ mmHg is by about 5 mmHg lower than

IOP, which means that this deviation cannot be linked to the shell's bending rigidity. Inequality $IOP_G < IOP$ above 16 mmHg, in the nonlinear model is as contrary to our intuition as in the linear model. But in the latter model the numerical solution does not raise doubts as to its quality since the fact that the graph is so radically different from that of function (1) cannot be due to only calculation errors. Intuition in mechanical problems is a poor adviser.

5.3. Corneal profile

An interesting application of the numerical model to structural identification is corneal profile geometry. The corneal profile is often approximated with a circle whereby the cornea's outer surface is then treated as a segment of a sphere. But the spherical aberration caused by this shape of the lens and the results of topographic examination of the cornea, indicating that the curvature radius decreases as the distance from the eye's optical axis increases, speak against the sphere. Easier to use and better fitting the geometry of the cornea is the ellipse [17]

$$z(x) = \frac{1}{e^2 - 1}\left[\sqrt{R^2 + x^2(e^2 - 1)} - R\right] \tag{2}$$

changing its shape depending on eccentricity e from $e = 0$ (a circle), through $e = 1$ (a parabola) to $e > 1$ (a hyperbola). The parabola has a particularly desirable analytical form since being a 2nd degree polynomial it is easy to differentiate and integrate. Is it acceptable to approximate the corneal profile with the parabola in studies of cornea model optics?

One of the few studies devoted to this problem is [18] in which an attempt is made to determine the optimum eccentricity of the ellipse, used to describe the cornea's outer and inner outline in the linear-elastic model. The optimization was performed with regard to a peculiar aspect of the model, called optical self-adjustment. One should note that the type of function approximating the outer profile of the cornea has a strong influence on the model's optical system and its dynamics as the model parameters (*e.g.*, IOP or the radius of corneal apex curvature) are changed. The model's optical focus not only should be located near the fundus of the eye at the nominal value of IOP but also changes in its location, dependent on the fluctuations in IOP, are governed by strict rules. Not every function can meet their requirements.

The optical self-adjustment of the eye is a hypothesis advanced by KASPRZAK [19]. As applied to the real eye, it reads as follows: the quality of the image on the retina of an unaccommodating eye does not depend on IOP. Physiological deviations of IOP from the mean value vary depending on the time of the day, the blood pressure, the body position and many other circumstances, including the health ones. The deviations do not usually exceed 5 mmHg. Of an eyeball model the Kasprzak hypothesis requires that its performance should be relatively easily verified: intraocular pressure fluctuations around the nominal value, with an amplitude of at least 30% of this value, should not have a noticeable effect on the location of the optical focus relative to the fundus while the lens should retain constant focal power. This model performance is hardly likely if the model is not structurally adapted for this purpose. When the pressure is increased, the eyeball expands – the cornea displaces forward while the corneal apex curvature radius increases. As a result of the stiff displacement of the cornea, the focus shifts forwards whereas the increase in the apex curvature radius shifts the focus backwards. When the numerical model's structural (geometrical and material) parameters are properly matched, the absolute values of the focus shifts are equal to each other and ultimately the location of the focus does not change. Such a model is optically self-adjusting.

In the discussed paper [18] the self-adjustment of the eyeball model was achieved through a properly matched limbus ring stiffness. The calculation results showed that limbus stiffness depends monotonically on the corneal profile ellipse eccentricity in the self-adjusting model. Initially, at an eccentricity close to zero, the model shows almost spontaneous self-adjustment. In order to increase eccentricity, it was necessary to increase the tension stiffness of the limbus ring. Initially, increments in limbus ring stiffness were small but once $e = 0.5$ was exceeded, they rapidly grew and at e close to 0.65 the limbus stiffness required to retain self-adjustment approached infinity. This result does not depend on the cornea's Young modulus, provided the ratio of the sclera modulus to the cornea modulus amounts to 5 (this value is justified by both experimental results [3] and physical predictions [13]).

Experiments of this kind show the strategy of identifying eye structures, based on the numerical model, to be highly effective. The obtained result imposes clear limits on the function used to describe the cornea's topography. In particular, it provides an answer to the question asked above – the parabola (the more so the hyperbola) is unsuitable for corneal profile approximation in the linear model.

6. Conclusions

At present, experimental results which could be used to create a numerical model of the human eyeball are far from satisfactory. The results of strength tests carried out on specimens excised from eye tissues and on whole eyeballs differ too much to be a reliable source of data. The technique of identifying the material of the eye's shells, consisting in the numerical simulation of clinical tests (such as applanation tonometry) turns out to be more effective.

Numerical eye models, even though they prove to be suitable for the purpose, are still far from perfect, particularly when applied to predict the results of refractive surgery. The cause is not so much the lack of a method, but rather faulty calculation programs. The resulting solution errors are due to, at least partially, the controversies around material constants and the associated simplifications.

The identified experimental and numerical shortcomings contribute to the persistence of conflicting beliefs about solutions achievable today. The results obtained by the author suggest that the principal equation of applanation tonometry (the Imbert–Fick law) cannot be satisfied by the real eyeball, even when the latter has calibration dimensions, since the law is based on false assumptions.

The shortcomings also affect the other aspects of the eyeball model's structure and behaviour – rheological material parameters, accommodation and fixing in the eye socket, *i.e.* the boundary conditions. All have an effect on the model's optical functions. Therefore one can conclude that the problem lies in the too little weight attached to the correctness of assumptions and solutions. The investigation of the optical system of the eyeball by means of a numerical model constitutes a new quality in mechanics and requires a new approach to eyeball design. The eye is not a mechanical structure in the classical sense – its function is not to carry loads, but to see.

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