

# Formulized analytical technique for gain characteristics of phosphate glass Er<sup>3+</sup>/Yb<sup>3+</sup> co-doped waveguide amplifiers

YU-HAI WANG, CHUN-SHENG MA\*, DE-LU LI, DA-MING ZHANG

State Key Laboratory on Integrated Optoelectronics, College of Electronic Science and Engineering,  
Jilin University, Changchun 130012, People's Republic of China

\*Corresponding author: mcsheng@163.com

Novel formulas for analyzing the gain characteristics of the phosphate glass erbium-ytterbium (Er<sup>3+</sup>-Yb<sup>3+</sup>) co-doped waveguide amplifier (EYCDWA) are derived from the rate equations and the light propagation equations under the uniform dopant and steady-state conditions. In the derivation of these formulas, we have neglected the amplified spontaneous emission (ASE) and have introduced the initial energy transfer efficiency. By using these formulas, the effects of the pump power, signal power, dopant concentration and waveguide length on the gain characteristics of the EYCDWA are analyzed, the comparison is performed between the EYCDWA and the singly erbium-doped waveguide amplifier (EDWA), and some useful results are obtained.

Keywords: waveguide amplifier, erbium-ytterbium co-dopant, rate equations, propagation equations, gain characteristic.

## 1. Introduction

The erbium-doped waveguide amplifier (EDWA) has attracted more and more interest in optoelectronic integrated circuits (OEICs), because of its larger signal gain in a small device size compared with the singly erbium-doped fiber amplifier (EDFA) [1–6]. It has been reported that the EDWA has a shortage as follows. One excited erbium ion transfers energy to the other excited ion, causing the acceptor to be promoted to a higher energy state and the donor to be deexcited to the ground state nonradiatively, which would enhance processes of up-conversion by energy transfer [7]. In order to enhance the gain of unit length, an EDWA requires high erbium ion (Er<sup>3+</sup>) concentration. However, high Er<sup>3+</sup> concentration will increase the number of the Er<sup>3+</sup> clusters, and hence reduce the spacing between the Er<sup>3+</sup> ions. In this case, the overlapping between the electron clouds of Er<sup>3+</sup> ions becomes severe, which causes the energy transfer between the Er<sup>3+</sup> ions, increasing the excited state absorption

(ESA). Therefore, the clustering enhances the ESA [8]. The clustering greatly reduces the pump efficiency and degrades the gain performance [6, 9–12].

Fortunately, the rare-earth element ytterbium, as a sensitizer, exhibits a better overlapping between the  $\text{Yb}^{3+}$  emission spectrum ( ${}^2F_{5/2}$ – ${}^2F_{7/2}$ ) and the  $\text{Er}^{3+}$  absorption spectrum ( ${}^4I_{13/2}$ – ${}^4I_{15/2}$ ) and an intense broad absorption in the wavelength range from 800 to 1080 nm, and has a weak clustering effect and a large absorption cross-section compared to erbium, by which high ytterbium ion ( $\text{Yb}^{3+}$ ) dopant level can be realized in the waveguide. This can noticeably reduce the quenching side-effect caused by high  $\text{Er}^{3+}$  dopant concentration [13], so the erbium-ytterbium ( $\text{Er}^{3+}$ - $\text{Yb}^{3+}$ ) co-dopant can efficiently improve the gain characteristics of the waveguide amplifiers. The clustering would result in the process of the up-conversion of the energy transfer. The doping of  $\text{Yb}^{3+}$  ions can greatly separate the spacing of the  $\text{Er}^{3+}$  ions and prevent the interaction between the  $\text{Er}^{3+}$  ions, and hence efficiently reduce the number of the clusters [14]. Therefore, the phenomenon of up-conversion can be ignored when the high  $\text{Yb}^{3+}$  concentration is doped. This indicates that the  $\text{Er}^{3+}$ - $\text{Yb}^{3+}$  co-doped waveguide amplifier (EYCDWA) has more potential applications than the EDWA.

Though some numerical methods, such as the finite difference beam propagation method (FD-BPM) [15] and Runge–Kutta (RK) method [16], can be used to analyze the gain characteristics of the EYCDWA, the simulation process is complicated, and requires much time spent on computing. In this paper, first in order to simplify the simulation process, novel formulas for analyzing the gain characteristics of the EYCDWA are derived from the rate equations and the light propagation equations based on the energy transfer process of  $\text{Er}^{3+}$  and  $\text{Yb}^{3+}$  ions. Then, by using these formulas, the effects of the pump power, signal power, dopant concentration and waveguide length on the gain characteristics are analyzed for the phosphate glass EYCDWA. Finally, some useful conclusions are reached on the basis of the analysis and discussion.

## 2. Theory

In this section, we propose an analytical technique for investigating the EYCDWA. Figure 1 shows the energy levels of an  $\text{Er}^{3+}$ - $\text{Yb}^{3+}$  co-doped system [17, 18]. In terms of this figure, the transition and energy transfer of  $\text{Er}^{3+}$  and  $\text{Yb}^{3+}$  ions are described as follows.

$\text{Yb}^{3+}$  ions on the fundamental level  ${}^2F_{7/2}$  absorb the energy of the 980-nm pump light, and transit up to the excited level  ${}^2F_{5/2}$ , then quickly resonantly transfer their energy to nearby  $\text{Er}^{3+}$  ions on the fundamental level  ${}^4I_{11/2}$ , and then  $\text{Er}^{3+}$  ions transit up to the excited level  ${}^4I_{11/2}$ . Since this excited level  ${}^4I_{11/2}$  is unsteady,  $\text{Er}^{3+}$  ions quickly decay to the metastable level  ${}^4I_{13/2}$ . In this process, the population inversion is realized on the metastable level  ${}^4I_{13/2}$ . The lifetime of  $\text{Er}^{3+}$  ions on the excited level  ${}^4I_{11/2}$  is very short (~ns order) compared with that on the metastable level  ${}^4I_{13/2}$  (about 8–10 ms), therefore, the number of  $\text{Er}^{3+}$  ions on the excited level  ${}^4I_{11/2}$  is so small that it can be neglected. For the phosphate glass as a suitable host medium, large phonon

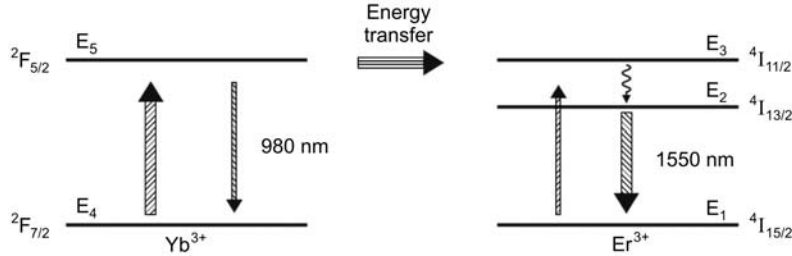


Fig. 1. Energy levels for the  $\text{Er}^{3+}$ - $\text{Yb}^{3+}$  co-doped system.

energy can urge the transition probability of  $\text{Er}^{3+}$  ions transiting from the excited level  ${}^4I_{11/2}$  to the metastable level  ${}^4I_{13/2}$ , which prevents the energy transfer from  $\text{Er}^{3+}$  ions back to  $\text{Yb}^{3+}$  ions [19].

Supposing  $N_1$  and  $N_2$  are the  $\text{Er}^{3+}$  ion concentrations on the  ${}^4I_{15/2}$  and  ${}^4I_{13/2}$  levels, respectively;  $N_{\text{Er}}$  is the total  $\text{Er}^{3+}$  ion concentration;  $N_4$  and  $N_5$  are the  $\text{Yb}^{3+}$  ion concentrations on the  ${}^2F_{7/2}$  and  ${}^2F_{5/2}$  levels, respectively;  $N_{\text{Yb}}$  is the total  $\text{Yb}^{3+}$  ion concentration. Under the conditions of the uniform dopant and the steady-state, the  $\text{Er}^{3+}$  ion and  $\text{Yb}^{3+}$  ion on the corresponding levels depend on the waveguide length  $z$ , *i.e.*,  $N_i = N_i(z)$ . Therefore, the multilevel rate equations for the  $\text{Er}^{3+}$ - $\text{Yb}^{3+}$  co-doped system are given by [20–22]

$$\begin{aligned} & \frac{\sigma_{12}(\nu_s) P_s(z) \Gamma_s}{A_c h \nu_s} N_1(z) + \frac{\sigma_{13}(\nu_p) P_p(z) \Gamma_p}{A_c h \nu_p} N_1(z) - \frac{\sigma_{21}(\nu_s) P_s(z) \Gamma_s}{A_c h \nu_s} N_2(z) - \frac{N_2(z)}{\tau_{21}} \\ & + \frac{\sigma_{45}(\nu_p) P_p(z) \Gamma_p}{A_c h \nu_p} N_4(z) - \frac{\sigma_{54}(\nu_p) P_p(z) \Gamma_p}{A_c h \nu_p} N_5(z) - \frac{N_5(z)}{\tau_{54}} = 0 \end{aligned} \quad (1)$$

with

$$N_1(z) + N_2(z) = N_{\text{Er}}, \quad N_4(z) + N_5(z) = N_{\text{Yb}} \quad (2)$$

where  $\Gamma_p$  and  $\Gamma_s$  are the overlapping factors of the pump and the signal, respectively;  $A_c$  is the area of the cross-section of the waveguide;  $\sigma_{12}(\nu_s)$  and  $\sigma_{21}(\nu_s)$  are the signal absorption and emission cross-section respectively;  $\sigma_{13}(\nu_p)$  is the pump absorption cross-section;  $\sigma_{45}(\nu_p)$  and  $\sigma_{54}(\nu_p)$  are the pump absorption and emission cross-section, respectively;  $h$  is Planck's constant.

Letting  $P_p$  and  $P_s$  be the pump and signal powers in the steady state, respectively, which satisfy the following transmission equations [22]

$$\frac{dP_p(z)}{dz} = -\Gamma_p \left[ \sigma_{13}(\nu_p) N_1(z) + \sigma_{45}(\nu_p) N_4(z) - \sigma_{54}(\nu_p) N_5(z) \right] P_p(z) \quad (3)$$

$$\frac{dP_s(z)}{dz} = \Gamma_s \left[ \sigma_{21}(\nu_s) N_2(z) - \sigma_{12}(\nu_s) N_1(z) \right] P_s(z) \quad (4)$$

## 2.1. Signal gain

From Equations (1)–(4), we obtain

$$\frac{1}{A_c h \nu_p} \frac{dP_p(z)}{dz} + \frac{1}{A_c h \nu_s} \frac{dP_s(z)}{dz} + \frac{N_2(z)}{\tau_{21}} + \frac{N_5(z)}{\tau_{54}} = 0 \quad (5)$$

By defining  $\eta_0 = \frac{N_2}{N_2 + N_5}$  as the initial energy transfer efficiency [23], that is,  $N_5 = \frac{1 - \eta_0}{\eta_0} N_2$ , and letting  $B = \frac{\tau_{21} \tau_{54}}{\tau_{54} + \tau_{21}(1 - \eta_0)/\eta_0}$ , Eq. (5) can be rewritten as

$$N_2(z) = -B \frac{1}{A_c h \nu_p} \frac{dP_p(z)}{dz} - B \frac{1}{A_c h \nu_s} \frac{dP_s(z)}{dz} \quad (6)$$

Setting  $S = \int_0^z N_2(z) dz$  and integrating Eqs. (3), (4) and (6), we get

$$S = \frac{\frac{1}{\Gamma_p} \ln \frac{P_p(z)}{P_p(0)} + \sigma_{13} N_{\text{Er}} z + \sigma_{45} N_{\text{Yb}} z}{\sigma_{13} + \frac{1 - \eta_0}{\eta_0} (\sigma_{45} + \sigma_{54})} \quad (7)$$

$$S = \frac{\frac{1}{\Gamma_s} \ln \frac{P_s(z)}{P_s(0)} + \sigma_{12} N_{\text{Er}} z}{\sigma_{12} + \sigma_{21}} \quad (8)$$

$$S = -B \frac{1}{A_c h \nu_p} [P_p(z) - P_p(0)] - B \frac{1}{A_c h \nu_s} [P_s(z) - P_s(0)] \quad (9)$$

From Equations (7) and (8), we get

$$P_p(z) = P_p(0) \left[ \frac{P_s(z)}{P_s(0)} \right]^\alpha \exp \left[ \alpha \Gamma_s \sigma_{12} N_{\text{Er}} z - \Gamma_p z (\sigma_{13} N_{\text{Er}} + \sigma_{45} N_{\text{Yb}}) \right] \quad (10)$$

where

$$\alpha = \frac{\Gamma_p \sigma_{13} + \Gamma_p (\sigma_{45} + \sigma_{54}) (1 - \eta_0) / \eta_0}{\Gamma_s (\sigma_{12} + \sigma_{21})} \quad (11)$$

Manipulating Eqs. (8), (9) and (10), we can arrive at the following equation

$$\left[G(z)\right]^\alpha \exp(-\alpha \Gamma_s \sigma N_{\text{Er}} z) = 1 - \frac{v_p P_s(0)}{v_s P_p(0)} \left[G(z) - 1\right] - \frac{\ln[G(z)] + \Gamma_s \sigma_{12} N_{\text{Er}} z}{B \Gamma_s (\sigma_{12} + \sigma_{21})} \frac{A_c h v_p}{P_p(0)} \quad (12)$$

with

$$G(z) = \frac{P_s(z)}{P_s(0)}, \quad \sigma = \frac{\sigma_{12} + \sigma_{21}}{\sigma_{13} + (\sigma_{45} + \sigma_{54})(1 - \eta_0)/\eta_0} \left( \sigma_{13} + \sigma_{45} \frac{N_{\text{Yb}}}{N_{\text{Er}}} \right) - \sigma_{12} \quad (13)$$

where  $G(z)$  is the gain of the amplifier.

## 2.2. Pump threshold

Letting  $G(L) = 1$ , from Eq. (12), we can express the pump threshold  $P_{\text{th}}$  as

$$P_{\text{th}} = P_p(0) = \frac{A_c h v_p \sigma_{12} N_{\text{Er}} L}{B (\sigma_{12} + \sigma_{21}) \left[ 1 - \exp(-\alpha \Gamma_s \sigma N_{\text{Er}} L) \right]} \quad (14)$$

## 2.3. Optimum waveguide length

When  $\left. \frac{\partial G(z)}{\partial z} \right|_{z=L_0} = 0$ , from Eq. (12), we can express the maximum gain  $G_0$  and the optimum waveguide length  $L_0$  as follows, respectively

$$G_0(L_0) = \gamma^{1/\alpha} \exp(\Gamma_s \sigma N_{\text{Er}} L_0) \quad (15)$$

$$L_0(G_0) = \frac{\ln G_0 - \frac{1}{\alpha} \ln \gamma}{\Gamma_s \sigma N_{\text{Er}}} \quad (16)$$

where

$$\gamma = \frac{A_c h v_p \sigma_{12}}{\alpha B \Gamma_s (\sigma_{12} + \sigma_{21}) P_p(0) \sigma} \quad (17)$$

Specially, when  $N_{\text{Yb}} = 0$ , then  $N_4 = N_5 = 0$ , in this case, the EYCDWA degenerates as an EDWA.

### 3. Analysis and discussions

In this section, we analyze the gain characteristics of the phosphate glass EYCDWA. The values of parameters used in the calculation are selected as [2, 22]: the pump wavelength  $\lambda_p = 980$  nm, signal wavelength  $\lambda_s = 1550$  nm,  $\text{Er}^{3+}$  absorption cross-section  $\sigma_{13}(\lambda_p) = 2.58 \times 10^{-25}$  m<sup>2</sup>,  $\text{Yb}^{3+}$  absorption cross-section  $\sigma_{45}(\lambda_p) = 1.0 \times 10^{-24}$  m<sup>2</sup>,  $\text{Yb}^{3+}$  emission cross-section  $\sigma_{54}(\lambda_p) = 1.0 \times 10^{-24}$  m<sup>2</sup>;  $\text{Er}^{3+}$  absorption cross-section  $\sigma_{12}(\lambda_s) = 6.5 \times 10^{-25}$  m<sup>2</sup>,  $\text{Er}^{3+}$  emission cross-section  $\sigma_{21}(\lambda_s) = 9.0 \times 10^{-25}$  m<sup>2</sup>;  $\text{Er}^{3+}$  emission lifetime  $\tau_{21} = 10$  ms;  $\text{Yb}^{3+}$  emission lifetime  $\tau_{54} = 2$  ms; the initial energy transfer efficiency  $\eta_0 = 11.5\%$ ; the core refractive index  $n_1 = 1.52812$ , the cladding refractive index  $n_2 = 1.51$ , waveguide cross-section  $A_c = 4 \times 4$   $\mu\text{m}^2$ , the calculated overlapping factors  $\Gamma_p = 0.921$  and  $\Gamma_s = 0.795$ .

Figure 2 shows the curves of the gain  $G$  versus the pump power  $P_{p0}$ , where we take the signal power  $P_{s0} = 1$   $\mu\text{W}$ ,  $\text{Er}^{3+}$  ion concentration  $N_{\text{Er}} = 1.0 \times 10^{26}$  m<sup>-3</sup>,  $\text{Yb}^{3+}$  ion concentration  $N_{\text{Yb}} = 0, 2.0 \times 10^{27}$  m<sup>-3</sup>, and waveguide length  $z = 1, 2, 3$  cm. We find that the gain increases as the pump power increases. Under the condition of the EYCDWA and the EDWA having the same size, when the pump power is larger than a certain value, the gain of the EYCDWA is larger than that of the EDWA. This

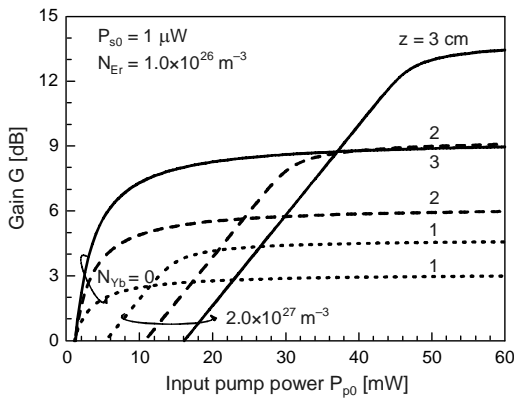


Fig. 2. Curves of gain  $G$  versus pump power  $P_{p0}$ .

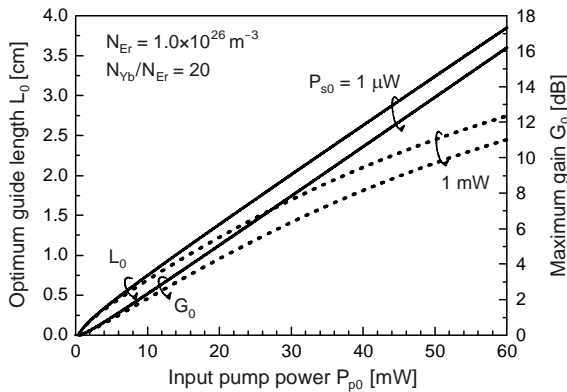


Fig. 3. Curves of optimum waveguide length  $L_0$  and maximum gain  $G_0$  versus pump power  $P_{p0}$ .

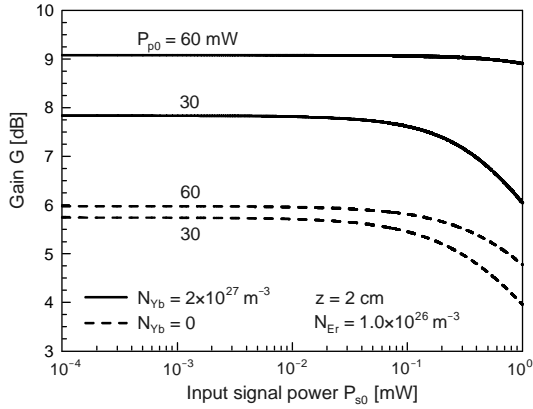


Fig. 4. Curves of gain  $G$  versus signal power  $P_{s0}$ .

is because the doped  $\text{Yb}^{3+}$  ions surround  $\text{Er}^{3+}$  ions and form  $\text{Er}^{3+}\text{-Yb}^{3+}$  ion-ion pairs, by which the  $\text{Yb}^{3+}$  ion absorbed photon energy transited phonon energy, which can sufficiently transferred to  $\text{Er}^{3+}$  ions to make more population reversion. This means that  $\text{Yb}^{3+}$  ions can provide an indirect exciting way to  $\text{Er}^{3+}$  ions. As the pump power increases to sufficiently large, almost all the  $\text{Er}^{3+}$  ions have realized the population reversion, then the gain becomes saturate. At the same condition, because the doped  $\text{Yb}^{3+}$  ions absorb some pump energy, the pump threshold power of the EYCDWA is larger than that of the EDWA.

Figure 3 shows the curves of the optimum waveguide length  $L_0$  and the maximum gain  $G_0$  versus the pump power  $P_{p0}$ , where we take the signal power  $P_{s0} = 1 \mu\text{W}$ ,  $1 \text{ mW}$ ,  $\text{Er}^{3+}$  ion concentration  $N_{\text{Er}} = 1.0 \times 10^{26} \text{ m}^{-3}$ ,  $\text{Yb}^{3+}$  ion concentration  $N_{\text{Yb}} = 2.0 \times 10^{27} \text{ m}^{-3}$ , that is  $N_{\text{Yb}}/N_{\text{Er}} = 20$ . We can see that as the pump power increases, both the optimum waveguide length and the maximum gain increase. The optimum waveguide length for a large signal power is shorter than that for a small signal power. Moreover, the maximum gain of the former is less than that of the latter.

Figure 4 shows the curves of the gain  $G$  versus the signal power  $P_{s0}$ , where we take the waveguide length  $z = 2 \text{ cm}$ ,  $\text{Er}^{3+}$  ion concentration  $N_{\text{Er}} = 1.0 \times 10^{26} \text{ m}^{-3}$ ,  $\text{Yb}^{3+}$  ion concentration  $N_{\text{Yb}} = 0, 2.0 \times 10^{27} \text{ m}^{-3}$ , and pump power  $P_{p0} = 30, 60 \text{ mW}$ . We can observe that within the range of the input signal power  $P_{s0} < 2 \times 10^{-2} \text{ mW}$ , the gain nearly keeps a constant for every curve. Beyond this, the gain decreases obviously with an increase in the input signal power. This is because stronger signal power can decrease  $\text{Er}^{3+}$  ion population reversion, and so the gain becomes weak.

Figure 5 shows the curves of the gain  $G$  versus the waveguide length  $z$ , where we take the signal power  $P_{s0} = 1 \mu\text{W}$ ,  $\text{Er}^{3+}$  ion concentration  $N_{\text{Er}} = 1.0 \times 10^{26} \text{ m}^{-3}$ ,  $\text{Yb}^{3+}$  ion concentration  $N_{\text{Yb}} = 0, 2.0 \times 10^{27} \text{ m}^{-3}$ , and pump power  $P_{p0} = 30, 40, 50, 60 \text{ mW}$ . We find that as the waveguide length increases, first the gain increases to a maximum, and then decreases. The maximum gain is corresponding to the optimum waveguide length. As an example, for the pump power  $P_{p0} = 30 \text{ mW}$ , the optimum waveguide length is about  $2 \text{ cm}$  and the maximum gain is about  $7.8 \text{ dB}$ . The larger the pump power, the larger the optimum waveguide length becomes. If the waveguide length is

too large so as to the gain less than zero, in this case, the  $\text{Er}^{3+}$  ion population reversion cannot be realized in the waveguide, moreover,  $\text{Er}^{3+}$  ions absorb the signal energy, and then the gain turns to the absorption. The introduction of  $\text{Yb}^{3+}$  ions can shorten the optimum waveguide length greatly, compared with the case of  $N_{\text{Yb}} = 0$  (EDWA); this is of benefit to the miniaturization and the integration of the EYCDWA device.

Figure 6 shows the curves of the gain  $G$  versus the  $\text{Yb}^{3+}/\text{Er}^{3+}$  ion concentration ratio  $N_{\text{Yb}}/N_{\text{Er}}$ , where we take the signal power  $P_{s0} = 1 \mu\text{W}$ , pump power  $P_{p0} = 40 \text{ mW}$ ,  $\text{Er}^{3+}$  ion concentration  $N_{\text{Er}} = 1.0 \times 10^{26}$ ,  $1.6 \times 10^{26} \text{ m}^{-3}$ , and waveguide length  $z = 2, 3, 4 \text{ cm}$ . We can see that the gain is negative when the ratio  $N_{\text{Yb}}/N_{\text{Er}} < 6.5$ . In this case, the negative gain is due to the small ratio  $N_{\text{Yb}}/N_{\text{Er}}$ ,  $\text{Yb}^{3+}$  ions absorb the pump energy insufficiently,  $\text{Er}^{3+}$  ion clusters may occur, which leads to concentration quenching, so the energy transfer from  $\text{Yb}^{3+}$  ions to  $\text{Er}^{3+}$  ions is not very efficient. Beyond this value of the ratio  $N_{\text{Yb}}/N_{\text{Er}}$ , the gain becomes positive; the gain increases nearly linearly, and then becomes saturate with further increasing of the ratio  $N_{\text{Yb}}/N_{\text{Er}}$ .

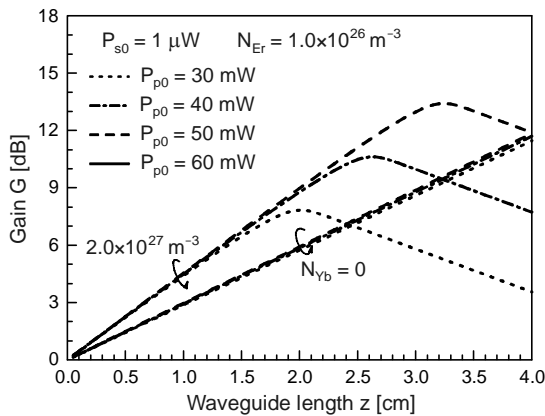


Fig. 5. Curves of gain  $G$  versus waveguide length  $z$ .

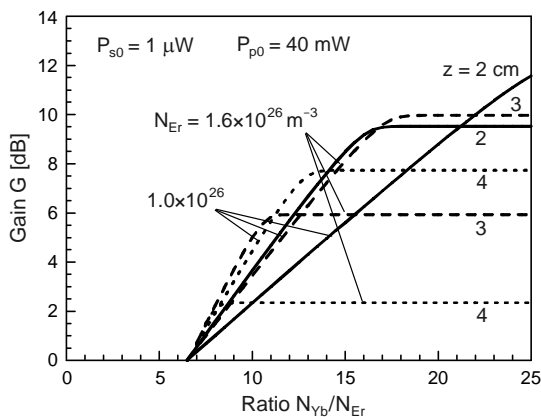


Fig. 6. Curves of gain  $G$  versus  $\text{Yb}^{3+}/\text{Er}^{3+}$  ion concentration ratio  $N_{\text{Yb}}/N_{\text{Er}}$ .



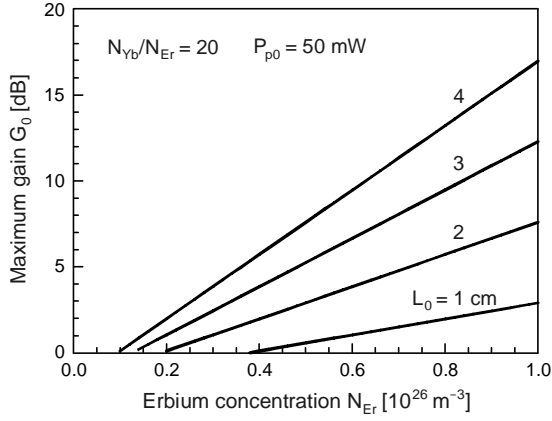


Fig. 7. Curves of maximum gain  $G_0$  versus  $\text{Er}^{3+}$  ion concentration  $N_{\text{Er}}$ .

The larger the  $\text{Er}^{3+}$  ion concentration, the faster the saturate gain is reached. The reason of the gain saturation is that, for a certain  $\text{Er}^{3+}$  ion concentration, when the ratio  $N_{\text{Yb}}/N_{\text{Er}}$  increases to a certain value, these  $\text{Er}^{3+}$  ions have all realized population reversion, therefore, the gain becomes constant, the gain saturation appears, and further increasing of the ratio  $N_{\text{Yb}}/N_{\text{Er}}$  no longer affects the gain characteristics.

Therefore, it is important to select an appropriate ratio  $N_{\text{Yb}}/N_{\text{Er}}$ . There is a definite optimal ratio  $N_{\text{Yb}}/N_{\text{Er}}$  for a certain waveguide length and a certain  $\text{Er}^{3+}$  ion concentration. For example, in the case of the waveguide length  $z = 3 \text{ cm}$ , pump power  $P_{p0} = 40 \text{ mW}$ , and signal power  $P_{s0} = 1 \mu\text{W}$ : when  $\text{Er}^{3+}$  ion concentration is  $1.0 \times 10^{26} \text{ m}^{-3}$ , the optimal ratio  $N_{\text{Yb}}/N_{\text{Er}}$  is about 18 and the corresponding gain is about 9.9 dB; while  $\text{Er}^{3+}$  ion concentration is  $1.6 \times 10^{26} \text{ m}^{-3}$ , the optimal ratio  $N_{\text{Yb}}/N_{\text{Er}}$  is about 11 and the corresponding gain is about 5.8 dB.

Figure 7 shows the curves of the maximum gain  $G_0$  versus the  $\text{Er}^{3+}$  ion concentration  $N_{\text{Er}}$ , where we take the pump power  $P_{p0} = 50 \text{ mW}$ , ratio  $N_{\text{Yb}}/N_{\text{Er}} = 20$ , waveguide length  $z = 1, 2, 3, 4 \text{ cm}$ . We can observe that the maximum gain increases as the  $\text{Er}^{3+}$  ion concentration increases. For a certain maximum gain, the shorter the optimum waveguide, the larger the  $\text{Er}^{3+}$  ion concentration required. Moreover, only as the  $\text{Er}^{3+}$  ion concentration increases to a certain value, the gain becomes positive, the device begins to have the amplification function. For instance, when the waveguide length  $z = 2 \text{ cm}$ , the  $\text{Er}^{3+}$  ion concentration must be larger than  $2 \times 10^{25} \text{ m}^{-3}$ .

#### 4. Conclusions

On the basis of preceding analysis and discussion for the gain characteristics of the phosphate glass EYCDWA, some conclusions are reached as follows.

The sensibilization of  $\text{Yb}^{3+}$  ions can effectively restrain the  $\text{Er}^{3+}$  ion clusters, and reduce up-conversion nonlinear side effect. This can increase the total gain and the unit length gain greatly; therefore, the performance of the EYCDWA is better than

that of the EDWA. Furthermore, the introduction of  $\text{Yb}^{3+}$  ions can shorten the optimum waveguide length; this is propitious to the miniaturization and the integration of the EYCDWA device.

The device has the maximum gain under the optimum waveguide length. When the pump power and the optimal  $\text{Er}^{3+}$ - $\text{Yb}^{3+}$  co-dopant ratio are sufficiently large, the gain tends to saturation. For different  $\text{Er}^{3+}$  ion concentration and waveguide length, there are different optimal  $\text{Yb}^{3+}/\text{Er}^{3+}$  ion concentration ratio. Therefore, it is important to select a proper value of this ratio to either increase the gain of the device, or avoid the co-dopant over large.

By using this proposed analytical technique for the EYCDWA, satisfactory results can be obtained conveniently, which are in agreement with those of the numerical simulation reported in a previously published paper [22], and the relative error is within 3%. So we think that the EYCDWA model considered is reasonable, and the technique presented is useful and valuable for the characteristic analysis, parameter optimization and structural design of this kind of waveguide amplifiers.

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